Graphics Calculator

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Package **MTH620A, *Graphics Calculator***

Computer Apple IIe, IIc, IIGS; 64K; one 5.25" drive

Package Contents
- One copy of the User's Manual for *Graphics Calculator*
- Software: two Program disks (one original and one backup); one Auxiliary/Demonstration Disk (unprotected)
- Software Purchase Agreement

Warranty Refer to the *Software Purchase Agreement* card included with the package for details of the warranty. To be sure that the disks you have received are not defective, test them immediately by following the "Starting Up" procedure below.

Backup You may not copy the Program Disk for this package, so we are including one backup copy. You are encouraged to copy the Auxiliary/ Demonstration Disk. One side holds the Auxiliary Disk files; the other holds the Demonstration Disk files. We recommend that you separate them onto two different working disks. First create a working copy of the Auxiliary Disk, then flip the disk over to the Demonstration Disk side and copy it to a second disk.

Starting Up Place the Program Disk in Drive 1. If the computer is off, turn it on, and the program will start automatically. If the computer is already on, start the program in one of the following ways.

Apple IIe or Apple IIc: press the RESET key while holding down the key and the CONTROL key.

Apple IIGS: press the RESET key while holding down the key and the CONTROL key. If you are unable to start the program, see "Changing the startup drive" in Appendix A of the Apple IIGS Owner's Guide.

Note: If you have difficulty starting the disk, refer to your Apple II Manual for DOS (Disk Operating System), consult a local Apple II expert, or call us at CONDUIT.

IMPORTANT: To test that the disk is functioning properly, continue the program past the initial screen. Test each disk individually before continuing to use the software.

Documentation These notes supplement the User's Manual for *Graphics Calculator* by Konemann, et al. Refer to the User's Manual for background reading and suggestions for using the program.

Your First Run After starting the program, simply follow the instructions on the screen. To test the Demonstration Disk, choose the second option on the Main Menu and wait for instructions to insert the Demonstration Disk. From the Demonstration, you can press the Esc key to begin using the *Graphics Calculator* program interactively. See the the section in Part I of the User's Manual "Interactive Use—A Quick Start" for a brief introduction to using *Graphics Calculator* interactively.

Making your own Auxiliary and Demonstration Disks You may make as many copies of the Auxiliary Disk as you wish for generating demonstrations, or for your own or student work disks. In fact, you can provide students with work disks which contain setups and demonstrations as illustrations or starting points for their assignments. See the sections "Loading and Saving Setups" and "Programming *Graphics Calculator*" for examples of how to make use of the Auxiliary Disk.

Note for IIGS Users This program modifies some of the Options parameters of the Control Panel. These parameters will be restored to their original values when you end the program according to the instructions on the screen, namely by inserting another disk and pressing RETURN. If the computer goes off while the program is running, the Control Panel will have the *Graphics Calculator* parameter values the next time the computer is turned on (see Appendix A, "The Control Panel Program," in the Apple IIGS Owner's Guide).

For Assistance If you have trouble starting the disk or using the programs, call CONDUIT at (319) 335-4100, and we will help you solve the problem.

Graphics Calculator

User's Manual

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Dedication

To Brenda and Joanne and more time together!

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Acknowledgements

We wish to thank those who have assisted in the development of *Graphics Calculator* over a period of more than three years. A special thanks to Quin Grosskopf, who class-tested early version of the software and made several suggestions for significant improvements. Many student assistants in the Mathematics and Computer Science departments at Carroll College contributed countless hours to typing, editing and proofreading several versions of this manual. Division secretary Mary Jane Landerr has done her usual excellent job on much of the final version. Student assistant Dane Black spent many hours proofreading the final version and checking manual examples against the software.

It has been a pleasure working with CONDUIT. In particular, Mathematics Editor Ken Daiker and his assistants have helped us a great deal. Their experience, patience, and helpful suggestions have resulted in important improvements in *Graphics Calculator* and in this manual.

Most of all, we thank our families for their continued support through three educational software projects over the past eight years. They encourage us when progress seems painfully slow, humor us when there is no progress, and celebrate with us when, by luck and hard work, results are better than expected.

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Preface

Teaching, learning, and doing mathematics are human endeavors. They require subtle communication, good judgement, creativity, inspiration, and emotion. Microcomputers cannot be programmed to have these qualities. It follows that they cannot teach, learn, or do mathematics by themselves. However, since microcomputers are very fast at calculating, plotting, storing, and retrieving, they can help us with mathematics. Computers can rapidly perform the more mundane tasks, leaving more time for us to enjoy the human aspects of mathematics. And since they are so fast, computers can change the way we think about and do mathematics. They let us use methods that would be impractical to carry out manually or with hand-held calculators.

Graphics Calculator takes advantage of the microcomputer's strengths. We have tried to make these strengths available in a format that is easy to begin using but that is sophisticated enough to remain useful. *Graphics Calculator* is not a single-purpose demonstration, tutorial, or drill and practice program. It is designed to help you quickly obtain, use, and communicate information about functions and equations so that you can do the more enjoyable part: experiment, be creative, make judgements, and communicate the results.

Integrated Design

Graphics Calculator can be used across the elementary mathematics curriculum, just as a hand-held calculator can be used. However, *Graphics Calculator* has more capabilities. The Graphics, Array, and Calculator displays are the main displays. You can define up to 8 functions and 15 parameters at once, then use them on all three displays to plot graphs, generate tables of values, and perform elaborate calculations. These features are integrated into a single program to produce a "function spreadsheet" that you can use to do significant elementary mathematics. There are additional features designed especially for teachers to use in classroom presentations.

Graphs and Coordinate Values

The Graphics display can plot graphs in a variety of ways on coordinate systems of your choice. It will enable you to plot function graphs, parametric equation graphs, and polar graphs in different colors and styles; project accurate coordinate values for *any* point on a graph; zoom in on important points; or trace domains onto ranges. You can obtain accurate values directly from graphs and use them for calculations on other displays.

Tables of Values

The Array display allows you to scroll through tables of 9-digit function or parametric equation values. Choose the function, the starting value, and the table increment; then scroll up or down through as many values as you need. Change the increment or starting value at any time. You can use the values obtained for plotting or calculating on other displays.

Integrated Programmable Calculator

You can use the Calculator display as a scientific calculator. Use up to 8 functions and 15 parameters at once. All BASIC intrinsic functions and arithmetic operations are available. You can use composite functions and logical expressions. There are facilities to program the Calculator so that it will repeat a sequence of operations automatically. Since the Calculator is linked to the Graphics and Array displays, you can use it to analyze or process values obtained from graphs or tables.

Optional Menus

The above features (and many more) are integrated into a single program with a simple control structure. Menus and help displays are optional. They may be avoided if you don't need them. You are not bound to a rigid sequence of operations as with some menu systems. You can select operations in any order you find appropriate. Typing is minimized because all functions and parameters that you define on one display are used on other displays. They can be changed without retyping the entire expression.

Save Setups

After you configure *Graphics Calculator* with the functions, parameters, variables, and coordinate system you want, you can save that setup on a disk. Then, whenever you need it, load and use the setup again. This allows you to switch rapidly and accurately between elaborate setups. It also saves a great deal of time and typing. You can interrupt a session at any point and later resume work exactly where you left off.

Print All Displays

Any display from *Graphics Calculator* can be printed. You can print graphics displays, tables of values, calculator displays, menus, catalogs of setups, or any other display. You can supply a title at the bottom of each display you print.

Classroom Uses

Teachers can use *Graphics Calculator* as a modern supplement to the chalkboard in mathematics classrooms. With a large monitor or video projection system, it provides scrolling tables of values, graphs, and calculations for the entire class to see.

Scrolling tables of function values save calculation time and help you discuss functions numerically. Corresponding graphs are plotted in seconds so that you can experiment with functions in class and show more examples. You can analyze graphs quickly because accurate numerical values are obtained directly from them. These can be used in further calculations on the Calculator display.

Prepared Demonstrations

Teachers can prepare and save elaborate setups for specific topics. In class, load setups as you need them, and use them interactively on all three integrated displays. There are also facilities to operate *Graphics Calculator* automatically by simply pressing the space bar. You can prepare demonstrations and step through them in class with a minimum of typing. You can build your own library of setups and prepared demonstrations.

A Modern Mathematical Tool

With *Graphics Calculator*, you can obtain accurate solutions to realistic problems. Its graphic/numeric techniques are intuitive and provide excellent supplements to traditional algebraic methods. They also introduce students to methods that are used when algebraic techniques fail. The computer does what it does best—calculate, plot, store, and retrieve—and you do what you do best: experiment, guess, make judgements, and draw conclusions. *Graphics Calculator* is a modern tool that helps you teach, learn, and do mathematics!

Part I: Beginning to Use *Graphics Calculator*

An Overview

The three main modes of *Graphics Calculator* are:

Calculator: A multiparameter, multifunction calculator
Array: A scrolling table of function values
Graphics: A versatile, fast graphics display with additional built-in features.

These three modes are fully integrated and use the same functions, variables, and parameters. In addition to the built-in BASIC functions, you can define up to six functions at once, then plot their graphs, scroll tables of function values, or calculate with them. You can use each mode by itself, or you can use all three modes together to solve a problem or study a key concept. In this sense, *Graphics Calculator* is a spreadsheet for mathematical functions.

As you read the first few sections of this manual, you will be calculating, scrolling function tables, and graphing almost at once. *Graphics Calculator* is extremely easy to begin using. Short, convenient menus are controlled by the ← and → keys. You can branch to Help screens at any time by pressing the ⌘-H key.

Although *Graphics Calculator* is easy for beginners, it is very versatile. You can begin using it as a simple function-graphing utility, a scrolling table of function values, or a scientific calculator. Soon you will be using several features together to solve difficult problems by combining graphic and numeric techniques.

You can use *Graphics Calculator* for iteration techniques. You can save your favorite setups for use later, and you can print all displays, including menus and catalogs.

Teachers can use *Graphics Calculator* to project graphs, tables of values, and calculator displays for the entire class to see. This makes an excellent supplement to chalkboard presentations. You can prepare elaborate demonstrations and presentations on specific topics. These demonstrations are saved on disks for later use. After a demonstration, use *Graphics Calculator* interactively to answer questions and continue class discussion. You can even interrupt a demonstration, use *Graphics Calculator* interactively to answer questions, and resume the demonstration exactly where you left off.

A Demonstration

The demonstration on your Demonstration Disk shows a variety of *Graphics Calculator* features and applications. The entire demonstration you see was created and saved, and is presented to you, using only the *Graphics Calculator* system. You can produce similar demonstrations.

To run the demonstration:

1. Choose "Run Demonstration" from the Menu.
2. Follow the instructions as they appear on the screen. To step through the demonstration, press the space bar whenever the @ prompt flashes at the lower right of the display.
3. At the end of the first demonstration, use the menu as prompted to see more demonstrations.

To end the demonstration early press Ctrl-Reset to restart the system; or press Esc, then ⌘-R to restart. (If you press Esc, you'll be linked to DEMO Help screens.)

Interactive Use—A Quick Start

This section of the manual is designed to acquaint you quickly with some of *Graphics Calculator's* main features, so that you can begin using it immediately. Later sections show detailed examples of many more features.

First you need to know how to use this manual, how to load the program, and how to find your way around after you load it.

Using This Manual

This manual should be used interactively with the program. We assume that you will work the examples in each section of Part I. These examples are selected to demonstrate important features and techniques. However, feel free to experiment as you go along. Part II has additional examples and applications you may wish to use.

To find details quickly, use the *Graphics Calculator* Quick Reference Card and Appendix 5. The Appendices cover various technical aspects of *Graphics Calculator;* use them as a reference.

We assume that you know some algebra and what the graph of a function is. You should also know BASIC syntax for algebraic expressions and BASIC intrinsic functions. If you do not know BASIC syntax, see Appendix 8 for a brief introduction. A list of BASIC intrinsic functions appears in Appendix 4 and on Help displays.

Starting *Graphics Calculator*

At the beginning of each session, you must start the main program. At the first *Graphics Calculator* menu, choose "Start Main Program."

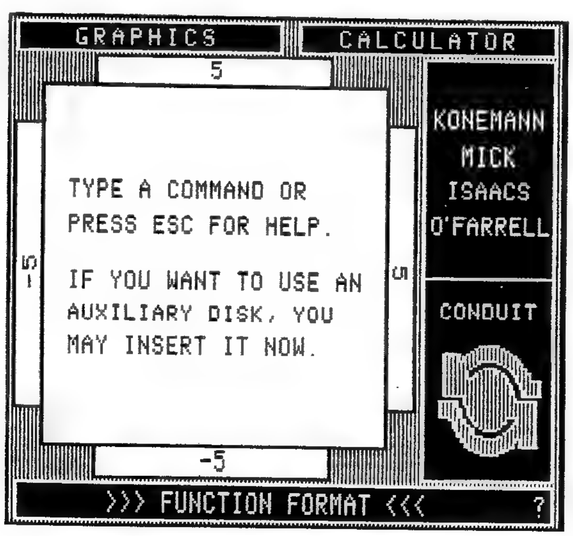

Figure 1.

The "?" Prompt

When the program is loaded, the display in Figure 1 appears with the flashing question mark, ?, at the lower right. This ? is an important prompt. It means you are in the interactive mode (rather than watching a demonstration), and that *Graphics Calculator* is waiting for you to make an entry. You can think of the ? prompt as "home base." From here, you can enter any other *Graphics Calculator* option.

Options at the "?" Prompt

The flashing **?** prompt appears on most displays. When it first appears, you hear the accompanying sound prompt. At the **?** prompt, you have just three options:

1. Press the Esc key to switch to a Help screen.
2. Press a single letter key (A, B, C, . . . , Z) to define a function or parameter.
3. Hold down the ♂ key and press a single letter to implement a command (♂-A, ♂-B, ♂-C, . . .).

These three options are introduced briefly in the following paragraphs. If you press keys other than these three options, the message "Invalid Key" appears, and the **?** prompt returns so that you can make another choice.

Esc to a Help Screen

The escape key, Esc, has two main purposes. One is to switch to the Help screens. If you press Esc at the **?** prompt on any display except Help, the Help screens are presented. (Pressing Esc in Help returns you to the previous display.) Press the Esc key several times now. Stop at the Help screens.

Use the cursor keys to cycle through the Help screens. Press ← or → to move backward or forward. Notice that these screens are numbered in the upper right-hand corner, and that the flashing **?** prompt is at the lower right. The first screen tells you how to use Help, and the second is an index to all Help screens.

In Case You Get Lost

You will also use the Esc key to exit commands or modes on any display and return to the **?** prompt. Then, if you press Esc again, the most recent Help screen appears. (This means you can't get lost.) If you don't know what to do next, press the Esc key several times. This will return you to a Help screen.

Defining Functions and Parameters

Pressing a single letter key begins the entry of BASIC expressions and numbers. These define the functions and parameters with which you want to work (option 2 above).

> *Example 1:* To begin this example, get to the flashing **?** prompt. If it is not there, press Esc repeatedly until you see it. Then, to define G(X), press the single key G . "G(X) = " appears, with a flashing cursor. Type X + 1 and press Return. You have just defined the function g(x) = x + 1.
>
> To define F(X), press F, type 2 * X – 1, and press Return. You have defined f(x) = 2x – 1. To define the parameter A, press A, type 3, and press Return. You have assigned a value of 3 to parameter A.

You may define functions and parameters on any display at the ? prompt. The definitions are then available for use on all displays.

Commands

After defining functions and parameters, you can use them for graphs, tables, and calculations. To do this, you will use command keys (Ó-A, Ó-B, etc.) to implement appropriate commands (option 3 above). For example, to issue the command "Graph Function," press Ó-G and Return. Try it. The graph of the last function you defined is plotted.

The *first-level commands* are listed with their corresponding options in Help screens 4 through 7. You may use any first-level command on any display when the ? prompt is present. You are not required to be at a Help screen. But if you need to see the Help screens at any time, press Esc at the ? prompt. Press Esc again to return to where you were.

The first-level commands are also listed on your *Graphics Calculator* Reference Card and in Appendix 6. The *second-level commands* listed there become valid only after a first-level command has been invoked. Second-level commands are introduced later in this manual as needed.

Before continuing with functions and parameters, you need to know more about first-level commands. At the same time, you will learn more about Help screens and about *Graphics Calculator's* three modes.

Help Screens

The Help screens are designed to provide lists of options you can use at the ? prompt, along with brief introductions to help you use them. These screens provide summaries of useful information, such as lists of special keys and BASIC intrinsic functions. They also provide information on changing entries and using Projections.

You have learned above how to access the most recently used Help screen with the Esc key and how to cycle forward and backward in these displays with the cursor keys. You can also go directly to a Help screen of your choice using the Ó-H, Help command. The following example illustrates this procedure.

Example 2: To choose a Help screen, press Ó-H at the ? prompt. The following menu appears:

```
HELP SCREEN:  [1]
```

Type the number of the Help screen you want, and press Return. When the Help screen you selected appears and the ? prompt returns, you may make any valid entry, including but not limited to those described on the Help screen. Press the cursor keys to cycle through more Help screens. You may also press Ó-H again to select a Help screen directly. Try this several times now. Note that Help screen 2 is an index to all other Help screens, Help screen 3 lists special keys and their uses, and Help screens 4 through 7 describe commands and options.

Press Esc to return to the display from which you started.

The Cursor Keys

Issuing a command at the ? prompt usually presents a one-line menu at the bottom of the display. This menu gives the name of the corresponding command and/or options for implementing the command. The ← and → cursor keys are often used to select options. To execute a selection, just press Return.

The next example uses ⌘-M to introduce *Graphics Calculator's* three main modes; it also illustrates the use of the ↑ and ↓, as well as the ← and → cursor keys.

Three Modes

The three main modes of *Graphics Calculator* are the Graphics, Array, and Calculator modes. It is easy to change modes, as shown in the following example:

Example 3: Press ⌘-M to initiate the Mode command. Find the following options menu at the bottom of the screen:

MODE: GRAPHICS ARRAY [CALCULATOR]

Press Return to select Calculator, and the Calculator display appears. Notice that all available functions and parameters appear here. Press ⌘-M again, and use the cursor keys to choose Array. When you press Return, the Array display appears with the same six functions you saw on the Calculator display. The table of values is empty now. Let's fill it using a few keystrokes which will be explained later. Press H, then type X^2 and press Return. On the screen, you will have:

H(X) = X^2

Press ⌘-A and Return. A table of values for X and H(X) will appear, with X = 0 and H(X) = 0 highlighted. Hold down the ↑ key until the table begins to change (scroll). The same thing will happen—but in the opposite direction—when you hold down the ↓ key. Press Esc to return to the ? prompt.

Finally, press ⌘-M, and select Graphics. When you press Return, the Graphics display appears. Any graphs previously plotted are still there.

In the following sections, the Graphics, Array, and Calculator modes are introduced in more detail.

The Calculator Mode

To use the Graphics, Array, and Calculator modes, you must enter BASIC expressions that define functions and parameters. This topic is best continued in Calculator mode. So, press ⌘-M, select Calculator, and press Return.

More Functions and Parameters

Each letter of the alphabet except T, Y, and Z appears in one of the three tables on the Calculator display. The top table is the function table, and the two middle tables are parameter tables.

To define a function or parameter:

1. Press the corresponding single letter.
2. Type the defining BASIC expression at the bottom of the display using correct BASIC syntax.
3. Press Return to complete the entry.

Example 4: We assume you are in the Calculator mode. To define the function $f(x) = 3x^2 - 4$, press F first. When "F(X) = " appears at the bottom of the display with a flashing cursor, type 3 * X^2 – 4. Finish the entry with Return. The new F(X) replaces the previous F(X) in the table.

For functions, the independent variable is always X. However, any parameter or other function appearing on the Calculator display can also be used in defining expressions. For example, you may use F(X) to define H(X) = F(X) + 2 * x.

> **If you have experience with BASIC, you will notice that using F(X) rather than FN F(X) is an exception to BASIC syntax.**

Function definitions are automatically checked for correct BASIC syntax . If the syntax is acceptable, the definition is recorded in the function table. If not, you have a chance to correct it. For mistakes, use the left-pointing cursor key (←) to backspace, and correct by retyping. Use the space bar to erase characters. (More elaborate procedures for making and changing entries are discussed in detail in Appendix 2.)

Example 5: To assign a value to the parameter A, first press A . When "A = " appears with a flashing cursor, type 2 and press Return. The value 2 is immediately recorded in the parameter table. To change it to 3/4, press A, type 3/4, and press Return. The decimal equivalent, .75, is recorded.

Parameters can be used in function definitions, and functions can be used in parameter assignments.

Example 6: First, press A, type 4 (followed by two spaces to erase the rest), and Return. Now A has value 4.

The function F(X) = 3 * X^2 – 4 was defined in Example 4. Now, assign the value A + F(A) to B. To do so, press B, type A + F(A), press Return, and note that the current

value of B is 48. [Why ? Because, A = 4 is used to calculate the value of A + F(A) which is assigned to B.]

Now define G(X) = A * X + B and assign C = G(–10). Since A = 4 and B = 48,

G(–10) = 8. This value is assigned to C. Next, assign D = A + C, and observe that D = 12 is recorded.

Current Values and Expressions

As you observed in the last example, each time you press Return to enter an *expression* for a parameter, the current *value* of that parameter is updated. The new current value is calculated from the defining expression and is displayed in the parameter table. The new expression is saved to be changed next time. If the expression is not changed before you press Return, the current value is still updated using the unchanged expression. This allows iteration with little typing, as we see next.

Iteration

Here is an example showing how to use expressions in an iteration process.

Example 7: First, define A = 0 and B = 1 to initialize these values. Then define A = A + 1 and B = A * B. If you repeatedly press A, Return, B, Return—in this order—the value recorded for B each time is A! [= A (A – 1) (A – 2) . . . 1; A factorial]. Try it.

Reviewing Expressions

If you press Esc rather than Return to complete an entry, neither the current value nor the current expression is changed. This allows you to check current expressions without updating and is one of the most frequent uses of the Esc key. For example, press B, Esc, and note that B does not change.

You can also review all current expressions at once. Press ⌘-S, Return to see the expressions. Then press Esc to return to the Calculator.

Entering and Changing Expressions

In *Graphics Calculator,* it is easy to change entries previously made. When you press a single letter, the last expression entered for that letter is presented for you to modify. (You also have the option at this point of pressing Esc, to leave the last expression intact.). After you make your changes, press Return to enter and implement the expression. The input routine has commands for deleting, inserting, and truncating expressions. (For more detailed information on the input routine, see Appendix 2, *Entering and Changing Titles, Filenames, and Algebraic Expressions.*)

Global Functions, Variables, and Parameters

The functions and parameters you have seen and defined on the Calculator display are also used in the Graphics and Array displays. You can change input in the same way on all displays, and defining a function or parameter on any display changes its definition on all displays. The current value of a parameter is also updated everywhere. The *variables,* X, Y, and T, also have the same values on every display.

Example 8: On the Calculator display, define F(X) = M * X + B, M = 1, and B = –2. Then use ⌘-M to change to the Graphics mode. When the Graphics display appears, press F again, and notice that F(X) = M * X + B appears just as it was defined on the Calculator display. At this point, the definition of F could be changed, but just press Return to define F with no change. This redefinition also displays the current values, M = 1 and B = –2, in the table at the right of the screen.

The Graphics Mode

The Graphics display lets you analyze functions graphically. You can compare two or more of them at one time to determine points of intersection. You can also get accurate approximations to maxima, minima, and and intercepts.

Graphing Functions

To graph a function, use ⌘-G at the ? prompt. The menu that appears is:

GRAPH FUNCTION: [F] G H R S Q

The ← and → cursor keys are used to choose which of the six functions to graph.

Example 9: To graph F(X) = M * X + B with M = 1 and B = –2, as defined in the previous example:

Press ⌘-G to begin the Graph Function command,

select F with the cursor keys, and

press Return to implement the command.

The graph of F is plotted using the current values of M and B. Now change the value of B to –1. (Just press B and enter –1.) Note that the current value of B is updated in the right-hand table. Now repeat the ⌘-G command to plot the new graph of F. Next, change M to –1, and plot F again.

The values of M and B can be changed repeatedly to get new graphs without redefining F(X) = M * X + B. This is a convenience when studying families of functions such as quadratic functions, $g(x) = ax^2 + bx + c$, or trigonometric functions, $h(x) = a \sin(bx) + c$. It is easy to investigate the effects on graphs of changing the coefficients a, b, c, . . .

Projecting Values

The Project/Plot command is one of the most important features of the Graphics display. It can be used to find coordinates of points on the graph of a function, and it can also be used to plot the graph. Accurate approximations of intercepts, points of intersection, and extreme values are available. Project/Plot has many other uses as well.

Example 10: We assume that F(X) = M * X + B is still defined, as in Example 9, with M = –1 and B = –1. Now, to erase the current graphs, press ⌘-E, select GRAPHS, and press Return.

To project values:

press ⌘-P to start the Project/Plot command,

select ARROWS from the first menu,

press Return to enter ARROWS,

select the function F from the second menu, and press Return again.

Immediately, a white point appears on the Graphics display, small arrows point to the X- and Y-coordinates, and these coordinates appear at the top of the display. The ← and → cursor keys are used to move this point and change its coordinates. Press one cursor key a few times. Notice that the point moves, the coordinate values displayed at

the top change, and small arrows move along the X and Y axes pointing at the X and Y projections. At the same time, a portion of the graph of F is plotted by connecting points.

Now hold down either the ← or → cursor key. The point moves more quickly because its coordinates are not displayed until the key is released. Move the point across the entire range of X-coordinate values to plot the graph. Then move it back to the coordinates X = 2 and Y = –3, and press Esc to end the Project/Plot routine.

The Graphics display has many more useful graphic/numeric features not mentioned in this brief introduction. You can zoom in on points and estimate their coordinates to 9 significant digits. Methods for changing coordinate systems, color and style of graphs, values projected, and many more features are explained later, when the Graphics display is considered in more detail. But first, try the Array display.

The Array Mode

The Array display has a scrolling table of 11 function values. It displays the same functions and variables as the Calculator and Graphics displays.

Example 11: We assume you have just completed Example 10. If not, repeat that example now. Before leaving the Graphics display, note the values of X and Y at the top of the display. You left them at X = 2 and Y = –3 in the Project/Plot routine. To go to the Array display for F(X) = M * X + B, press Ó-A at the **?** prompt, choose F, and press Return. (See Figure 2.)

```
F(X)=M*X+B
G(X)=0
H(X)=0
R(X)=0
S(X)=0
Q(X)=0

    X                   F(X)
-------------------------------
.75                 -1.75
1                   -2
1.25                -2.25
1.5                 -2.5
1.75                -2.75
2                   -3
2.25                -3.25
2.5                 -3.5
2.75                -3.75
3                   -4
3.25                -4.25

          PRESS ARROWS TO SCROLL
```

Figure 2.

Notice that six functions are displayed at the top, just as on the Calculator display. The center values in the table, X = 2 and F(X) = –3, are the *same* as you just projected on the Graphics display. They are highlighted in reverse video. This reverse video also serves as a cursor for scrolling the table up and down using the ↑ and ↓ arrows. Press an arrow to see the cursor move.

Each time you press ↑, the value of X decreases (the cursor moves up). When you press ↓, X increases (the cursor moves down). When the cursor reaches the top or bottom of the table, the table scrolls. Try it. Just hold down one arrow key. You can scroll through as many values as you like.

To see a particular value of F(X), press X and change the current value of X to the value you want. When Return is pressed, the table is regenerated, with the new cursor value at the center. For example, press X and change X to 1. When you press Return, X = 1 and F(X) = –2 become the values at the position of the cursor.

The default X-increment is Delta-X = .25. This is the amount that X changes each time a cursor key is pressed. The value of Delta-X is easily changed. Just press Ó-X and change the value of Delta-X that appears. Try changing it to .1, for example. When

Return is pressed, the table is again regenerated with X = 1 in the center, but the difference in X-values has changed from .25 to .1.

To exit the scrolling routine, press Esc, as usual. The **?** prompt returns to its lower right position, and you can use any first-level command again. For example, you can generate an array for a different function, redefine functions and parameters, change to Graphics or Calculator mode, or switch to a Help screen.

If you change parameters or define a function using parameters, the current value of each parameter used is displayed on the right side of the screen. For example, press F and Return to see the current values of M and B in the function F(X) = M * X + B.

Function Combinations

The Array display is used to create tables of nine-digit values for any function that you define in 30 or fewer characters. These same functions are used by the Graphics and Calculator modes. But since algebraic combinations like H(X) = F(X) + G(X) or H(X) = F(X) + A * SIN(B * X + C) are valid, even more complex functions can be used. Also, compositions of functions like H(X) = F(G(SQR(X))) are valid. Since six generic functions (F, G, H, R, S, and Q) are available in addition to BASIC's built-in functions (SQR, SIN, LOG, etc.), there is plenty of versatility. You can also use logical expressions like (X < 2) and ((X < 2) OR (X > 3)) to make conditional assignments. [When X is less than 2, then the expression (X < 2) has the value 1; otherwise, it is 0.]

Loading Setups from Disks

After you define a collection of functions and parameters and a corresponding coordinate system, you can save this setup on a disk and use it again later. This procedure is discussed in detail in Part II of this manual. Here, we want to introduce you to loading setups, because it will help us in later presentations.

There are several setups saved on the *Graphics Calculator* disk. The next example shows you how to find out which setups are saved on the disk and how to load them.

Example 12: The Setup command is used to load and save setups. Press ⌘-S to start this command. When you see the menu:

SETUP: [STATUS] LOAD SAVE CAT DEL PRINT

use the ← and → cursor keys to select CAT, then press Return. The catalog that you see lists the names of all the setups saved on the main program disk. Find the name "Quadratic" on the catalog.

Notice that the above Setup menu is still at the bottom of the screen. To implement the Load option, select LOAD, and press Return. When you see the NAME OF FILE prompt, type QUADRATIC and press Return. Press Return again to implement the ALL option from the menu:

LOAD: [ALL] FUNC PAR CMND

The Quadratic setup is now loaded.

To see the effects of what you have just done, switch to the Calculator mode. Notice that three functions—F, G, and H—have new definitions. To see what has happened to the parameter definitions, press ⌘-S, Return to see the Status display. The current expressions defining each parameter are displayed. Each function and parameter expression is followed by a brief label describing it. You might recognize the quadratic formula used for the zeros, I and J. Press Esc to return to the Calculator display.

To use this setup, you must assign values to the coefficients, A, B, and C. For example, press A, 1, and Return to assign A = 1. Then assign B = 7 and C = 6. To update the value of I, press I, Return. Do the same for J, U, and V.

To graph your quadratic function, press ⌘-G, Return. (You might want to press ⌘-E, select BOTH, and press RETURN to erase any previous graphs and values first.) Your graph should appear as in Figure 3. Notice that the scale on the coordinate system was changed when you loaded the Quadratic setup. To see the current values of A, B, and C, press F, Return to redefine F(X). You can change the values of A, B, and C here on the Graphics display and plot more graphs, or you can change to the Array or Calculator modes and use the same functions and parameters.

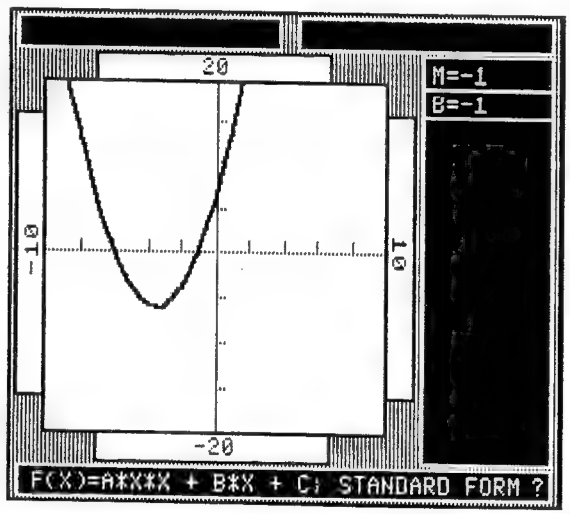

Figure 3.

Loading setups saves much typing and makes it easy to change from one detailed setup to another. You can also interrupt a session on *Graphics Calculator*, save the setup, and resume work later at the same point.

Parametric Format

To change to Parametric format, press ⌘-F, select PARAMETRIC, then press Return. *Graphics Calculator* restarts at the opening Graphics display, as if you had just loaded the program. However, now you are in Parametric format with the default setup. All function and parameter definitions are set to zero. To see this, press ⌘-M, Return to go to the Calculator display.

In Parametric format, eight functions of T are available: F(T), G(T), H(T), R(T), S(T), Q(T), X(T), and Y(T). The coordinates, X(T) and Y(T), are calculated as functions of the *independent variable*, T. On the Graphics display, the pair (X(T), Y(T)) is plotted. On the Array display, X(T) and Y(T) are listed in the table. The other six functions, F, G, H . . . , and the parameters A, B, C, . . . , are useful for defining X(T) and Y(T). Only X(T) and Y(T) are plotted on the Graphics display. Due to limited space, S(T) and Q(T) are not shown on the Array and Calculator function tables (but you can still use them).

If you are not familiar with parametric equations, the following examples will introduce you to them.

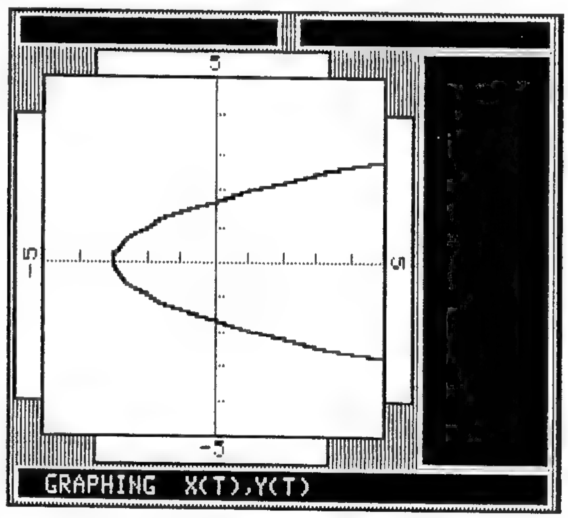

Figure 4.

Parametric Graphs

The Graphics display is controlled in Parametric format in very much the same way as it is in Function format.

Example 13: If you are not in Parametric format, use ⌘-F to get there. Then:

Press X, define x(t) = $t^2 - 3$ by typing T * T – 3, then Return.

Press Y, define y(t) = t by typing T, then Return.

Press ⌘-G, select YES, then Return.

Immediately, a parabola with horizontal axis is plotted. This is a graph of the parametric equations $x(t) = t^2 - 3$ and $y(t) = t$. If t is eliminated from the equations, the resulting equation is $x = y^2 - 3$. We recognize the parabola in Figure 4 as the graph of this equation.

Try ⌘-P, Project/Plot on this graph:

Press ⌘-P,
select ARROWS, and
press Return. Then,
press ← and → to project the coordinates of points on the graph.

Find the X and Y coordinate values at the top of the display. The X- and Y-values are calculated from T with your formulas, X(T) = T * T – 3 and Y(T) = T.

A T-interval and the T-value appear at the bottom of the display. Find the small arrow in the T-interval. It points to the current value of T, which you see at the right. Each time you press ← or →, the values of X, Y, and T change, and the small projection arrows move. If you hold down the ← or → key, the projections move quickly. When you release the key, current coordinates are printed.

Press Esc to exit Project/Plot.

Now try the Array display.

```
F(T)=0
G(T)=0
H(T)=0
R(T)=0

X(T)=T*T-3
Y(T)=T

   X(T)              Y(T)
--------------------------------
-1.4375          -1.25
-2               -1
-2.4375          -.75
-2.75            -.5
-2.9375          -.25
-3               0
-2.9375          .25
-2.75            .5
-2.4375          .75
-2               1
-1.4375          1.25

  T=0
```

Figure 5.

Parametric Tables

The Array display generates tables of values for parametric equations. You can compare the values of two functions in the same table. To see how, continue the last example.

Example 14: In the last example, you graphed the parametric equations

X(T) = T * T – 3, Y(T) = T. Now,

press ₲-A to begin the Array command,
select YES, and
press Return.

Immediately an array of values for X(T) and Y(T) appears. (See Figure 5.) The T-value at the bottom of the screen corresponds to the highlighted values of X(T) and Y(T) in the table. Use the ↑ and ↓ keys to scroll this table. Hold down an arrow key to scroll quickly. (You may also use ← or →.)

A specific value for T is assigned simply by pressing T, typing the value, and pressing Return. Similarly, the increment, Delta-T, used to generate the array is changed by pressing ₲-T, typing the value, and pressing Return. Try a few changes in T or Delta-T. Immediately after each change, a new array is generated .

To return to the **?** prompt, press Esc .

Polar Graphs

One important use of Parametric format is to graph polar functions. Another is to graph conic sections. For examples of polar graphs and conic sections, see the chapter in Part II called *Polar Graphs.* The Calculator operates much the same way in Parametric format as it does in Function format. Differences are discussed in Part II, in the section titled "Calculating in Parametric Format."

Summary

At the first *Graphics Calculator* menu, choose "Start Main Program."

At the flashing ? prompt, press Esc to switch to Help screens and back. Help screens 4–7 list all first-level commands and corresponding options.

At the ? prompt, press a single letter key to define the corresponding function or parameter. Use BASIC syntax for all arithmetic expressions. Press Return to complete the definition.

At the ? prompt, hold down the key and press a letter key to invoke the corresponding command. Use the ← and → cursor keys to make selections or enter data as prompted. Press Return to complete the entry.

The three main displays correspond to the three modes: Graphics, Array, and Calculator.

Make any entry at the ? prompt on any of the three main displays or on any Help screen.

To see a specific Help screen, press -H, enter its number, then press Return. Use ← and → to move backward and forward between displays, or press -H to choose another Help screen number.

Use the *Graphics Calculator* Quick Reference Card and the appendices of this manual to find information quickly. Detailed examples are presented in the next part of this manual.

Part II: Examples and Applications

Introduction

This part of the manual explores the detailed operation of each feature of *Graphics Calculator*. It presents examples which demonstrate specific applications of these features both singly and in combination.

Assuming that you have read and worked with the examples in Part I of this manual, you are generally familiar with *Graphics Calculator's* displays and terminology. After the first few examples in Part II, we will assume you know when to press the Return key, so that step is not mentioned explicitly thereafter, unless essential for clarity.

As you use this section of the manual, keep the Quick Reference Card handy for reminders. The appendices of this manual are also helpful for reviewing information quickly; Appendices 2 and 6 are particularly useful.

Explanations are presented first in terms of Function format. Details for Parametric format begin in the section *Function and Parametric Graphs, ⌘-F*. We begin with the definition of functions and parameters.

Defining Functions

A function may be defined on any display in which the flashing ? prompt appears. You may define any or all of the six functions F, G, H, R, S, and Q. To define one of these functions, press the corresponding letter and type the defining BASIC arithmetic expression, followed by Return. The independent variable is always X. Functions defined on one display may be used on all displays where the ? prompt appears.

If you have never worked with functions on *Graphics Calculator,* or if you wish to review, you should do Examples 6–11 in Part I of this manual before proceeding. Also, review Example 72 in Appendix 2 for help with making and changing entries.

Labeling Functions

You may use up to 30 characters to define a function. If you use fewer than 30 characters, you may use the remaining space for labels. Simply type a semicolon or colon, then use the remaining space for your brief label (as in the following example). Only the characters listed in Appendix 1 may be used in labels. If you press other keys, they will be ignored.

Example 15: Each of the following is a valid function:

(1) F(X) = A * X^2 + B * X + C ;QUADRATIC FUNCTION
(2) G(X) = (2) * X^2 + (-3) * X + (4)
(3) H(X) = U * F(X-V) + W ;TRANSLATION-MAGNIF.
(4) R(X) = X * (X < A) + (X^2) * (X > A) ;LOGICAL
(5) S(X) = LOG(1-X) + SQR(X) ;DOMAIN 0 < = X < 1
(6) Q(X) = SGN(X) * ABS(X)^(1/3) ;CUBE ROOT
(7) F(X) = SIN(N * P * X)

The first function is a general quadratic function and is so labeled. This form is useful for investigating the effects on graphs of changing coefficients. Each time the value of A, B, or C is changed, the new parameter value is used when a value of F(X) is calculated or when the graph is plotted. Note that the label QUADRATIC FUNCTION is preceded by a semicolon indicating a comment or label.

The second function, G(X), has explicit coefficients with space left for changing them. The input routine makes it very convenient to type in new coefficients without retyping the entire expression. (See Appendix 2 for details.) The spaces have no effect when the function is evaluated.

The third function, H(X), is useful in demonstrating the translations and stretching which occur on the graph of F(X) as the values of U, V, and W change.

The fourth function, R(X), has function value X if X < A, 0 if X = A, and X^2 if X > A. This is because the logical expressions (X < A) and (X > A) have value 1 when true and 0 when false.

The function S(X) has domain (0,1). But domains do not affect function definition. Only syntax is checked. If a value of S(X) is requested later with X not in the domain, you are informed that the value is undefined (see Appendix 6, *Error Messages*). While graphing, undefined values are ignored. The graph is plotted only on the domain for

which the BASIC expression has defined values—and then only on the part of the domain inside the coordinate window.

The next function, Q(X), is actually just $X^{1/3}$. We do not use X^(1/3) because it is not defined for negative numbers in BASIC. We use Q(X) to get a cube root function defined for negative X, too.

The seventh function, F(X), involves the parameters π and N. To generate the π symbol on the graphics display, press %. The parameter, P, has the same value as π and is used in text displays.

Function Displays

To review the definition of a function, press the corresponding letter at the ? prompt. If you do not want to change the definition, press Esc to return to the ? prompt. All six function definitions appear at the top of both the Array and Calculator displays. They also appear on the Setup/Status displays (see the section *Checking the Setup*, ♂-*S*). A function definition is also displayed when the function is graphed.

Function Symbols

Any valid BASIC algebraic expression with up to 30 characters may be used. For example, the functions F(X), G(X), . . . , Q(X) may be used to define other functions. [Note that F(X) is used, not FN F(X).] You may also use BASIC's built-in functions, such as COS, EXP, LOG, etc. See Appendix 4 for a complete list of BASIC functions.

Function Self-Reference

Self-reference (recursion) is not valid, directly or indirectly, in function definitions. For example, F(X) = F(X) + G(X) is invalid. The combination F(X) = G(X), G(X) = H(X) and H(X) = F(X) produces an error when H(X) is defined because of indirect self-reference. (Self-reference produces an "Out of Memory" error because the stack is filled.)

Composite and Logical Expressions

Composite functions are valid. For example, F(X) = SQR(G(LOG(X^2-1))) is acceptable. This allows you to experiment with functions and graphs you might not even attempt by manual methods.

Logical expressions like ((2 < X) AND (X < = 3)) may also be used. For example, the function

$$f(x)= \begin{cases} 2x + 3 & \text{for } x < 0 \\ x - 1 & \text{for } x \geq 0 \end{cases}$$

is defined in *Graphics Calculator* by

F(X) = (2 * X + 3) * (X < 0) + (X – 1) * (X > = 0).

Parameters in Function Definitions

There are 15 parameters represented by the letters A, B, C, D, E, I, J, K, L, M, N, O, U, V, and W. The parameter P is reserved for the ratio of the circumference of a circle to its diameter ($\pi = 3.14159265$). To produce the π symbol on the graphics display, press %. On text displays, % appears as P, but on graphics displays, it appears as the symbol π.

Each time a function is defined on the Graphics or Array display, the current value of each parameter used (except P) is printed on the right side of that display in the order used. If some parameter values—say, for M and B in F(X) = M * X + B—have been erased, simply redefine the function by pressing F and Return. The current values of M and B are printed again.

Each time a function value is calculated, these current parameter values are used in the calculation. The current value of a parameter may be changed any time the ? prompt appears. The value is updated on the display and used the next time function values are calculated. For an illustration, see Example 6 in Part I.

Since you have available six 30-character functions and 15 definable parameters (each of which may be defined by 30-character expressions), you can create very complex functions.

Defining Parameters

Parameters are defined in much the same way as functions. Fifteen parameters are available, represented by the letters A, B, C, D, E, I, J, K, L, M, N, O, U, V, and W. The parameter P is reserved for the ratio of the circumference of a circle to its diameter (π = 3.14159265). To produce the π symbol on the graphics display, press %. On text displays, % appears as P, but on graphics displays, it appears as the symbol π.

To define a parameter when the ? prompt is present, press the corresponding letter, type a valid BASIC expression, and press Return. The expression may contain numbers, functions, other parameters, and the parameter being defined. The current values of X, Y, and T may also be used, if you know them.

If you have not yet worked with parameters on *Graphics Calculator*, you should review Examples 4–7 in Part I of this manual before proceeding. Also review Appendix 2.

Current Values and Expressions

When you press Return to complete a parameter definition, the expression is first checked for correct syntax. If the syntax is valid, the expression becomes the *current expression* for that parameter. Then the value of the defining expression is calculated using the current value of each parameter and variable. The calculated value is displayed as the *current value* of the redefined parameter and is used in subsequent calculations.

If the calculated value is undefined (if, for example, the calculation results in division by zero), a message informs you that the current value has not been updated. The expression for the parameter is changed, however, if the syntax is correct. This allows you to enter an expression even when its current value is not defined. Later, when other parameters have been initialized, your current expression may be used in calculations that produce defined values.

The current value of a parameter may be updated without changing its defining expression. Simply press the correct letter. When the current expression appears for modification, do not change it; just press Return. The expression is evaluated using the current values of all parameters and variables. This value becomes the current value of your parameter. This method saves typing and allows for convenient iteration. See Example 7 for an illustration of iteration.

Parameter Labels

You may label parameter expressions just as you label function expressions. Simply type a semicolon to separate the defining expression from the label or comment that follows it. The following example is similar to Example 12 in Part I of this manual but requires more typing. You should compare these two examples.

Example 16: To analyze quadratic functions and inequalities, set up *Graphics Calculator* as follows:

First, use ⌘-R to restart with the default setup. This clears all functions and parameters to zero and sets up the default coordinate window. Then go to the Calculator display (use ⌘-M) and define:

```
F(X) = A * X^2 + B * X + C; QUADRATIC
A = 1    ; LEADING COEFFICIENT
B = -4   ; LINEAR COEFFICIENT
C = -5   ; CONSTANT TERM
D = B * B - 4 * A * C ;DISCRIMINANT
I = (-B + SQR(B^2 - 4 * A * C))/(2 * A); ZERO
J = (-B - SQR(B^2 - 4 * A * C)/(2 * A); ZERO
U = -B/(2 * A); X-VERTEX COORDINATE
V = F(U); Y-VERTEX COORDINATE
```

Each definition is followed by a label. These are optional and are used just as with function definitions. Simply follow the defining expression with a semicolon or colon, and use any remaining space for your label. Each line accepts 30 characters. Only those characters listed in Appendix 1 for Titles and Labels may be used in labels. Other characters are ignored.

As you press Return to complete the definitions of D, I, J, U, and V, the discriminant, zeros and vertex coordinates of F(X) are displayed as the current values of the respective parameters.

Notice that for A, B and C, some space has been left to change values later. For example, define A = .5, B = 2, and C = –5/4. This updates the coefficients for F(X). Now, press I and Return to update the current value of I. Do the same for J, U, and V. In this way, you can find the zeros and vertices of quadratic functions simply by changing the values of A, B, and C and updating I, J, U, and V.

If you would also like to see the corresponding graph, press ⌘-G, select F, and press Return. If you prefer, you can change coefficients and update I, J, U, and V on the Graphics display; or you can switch back to the Calculator display to update.

Now that you have taken the time to enter this elaborate quadratic setup into *Graphics Calculator*, you might want to save it for future use. To learn how, see Example 53 in the section *Loading and Saving Setups, ⌘-S.*

More examples of function and parameter definition and use appear in the sections of this manual devoted to the Graphics display and the Calculator display. These sections also illustrate the use of functions and parameters in command expressions.

Parameter Self-Reference

Unlike functions, parameters may be used in their own definition. Self-reference is valid. For example, A = A + B is valid. The current value of A is added to the current value of B, and the result is assigned as the new current value of A.

Example 17: The following sequence of operations produces terms in the well-known sequence of Fibonacci numbers:

Press ₫-R to restart and ₫-M to select Calculator mode.

Press A, define A = 1,

Press B, define B = 2,

Press A, define A = A + B,

Press B, define B = B + A,

Press A, Return, B, Return in this order several times.

Each time you press Return, the value of A or B is the next term in the Fibonacci sequence.

Other Parameter Uses

The use of parameters is not restricted to defining functions, other parameters, and calculations. Parameters are also used to store and pass values among various routines. In particular, many numeric values are used in commands to set up and operate features of *Graphics Calculator*. Each of these may be defined and updated using functions and parameters. This topic is discussed in more detail in the section *Checking the Setup, ₫-S.* Also see Examples 21 and 22.

Parameter Displays

The current value of each parameter appears on the Calculator display. To see all current values simultaneously, press ₫-M at the **?** prompt and choose "Calculator."

Each time a parameter is defined on the Graphics or Array displays, its current value is printed on the right side of the display. If a question mark (?) appears as the last character in a value, the value has been left-justified and *truncated* on the right to fit the display. For example, A = 1.23456E –10 may appear as A = 1.234? Care is needed in interpreting current values with a trailing "?" on the Graphics and Array displays. When in doubt, check the Calculator display for the full 9-digit value.

The current defining expression for each parameter is displayed each time the parameter letter is pressed at the ? prompt. To see an expression without updating the current value, press the desired letter, then press Esc .

To check all current expressions—including parameters, functions, variables, and command expressions—press ₫-S and Return to see the Setup/Status displays. These also show the current values of the command expressions. You may notice that if a number alone has been used to define a parameter, the current expression and the current value are essentially identical.

The Graphics Display

The Graphics display is a rapid function-grapher with many analytical features. With it, you can:

- easily change coordinate windows and scales;
- quickly magnify sections of graphs;
- conveniently project coordinates of important points on graphs and save them for later use;
- emphasize graphs or sections of graphs by plotting in various colors and/or line styles; and
- see the trace of the domain of a function onto the range to help you visualize functions as mappings.

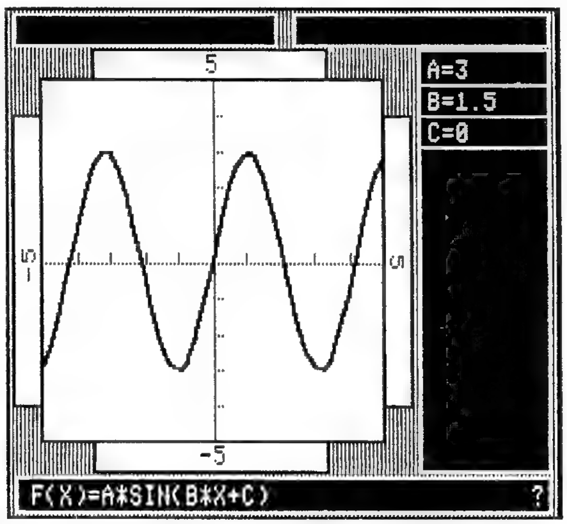

Figure 6.

Graphing Functions, ⌘-G

To plot a graph in the quickest way, press ⌘-G for the "Graph Function" command, select a function—F(X), for instance—with the ← and → cursor keys, and press Return. At each of a number of equally spaced points, a value of F(X) is calculated. Then the corresponding point on the graph is plotted and connected to the previous point plotted.

There are some exceptions. If the X-value is not in the domain of the function, as defined in BASIC, the computer speaker clicks, and the next value is attempted. If the calculated point is not inside the current coordinate window, it is not plotted, but a small arrow points to its position outside the coordinate window, and a sound is produced. Segments connecting calculated points are "clipped." That is, only the portions of the graph inside the current coordinate window are plotted. It is also possible to plot points without connecting them.

Example 18: First press ⌘-R to return to default status. Then,

define F(X) = A * SIN(B * X + C), A = 3, B = 1.5 and C = 0,
press ⌘-G for the "Graph Function" command,
and select F.

The graph is plotted quickly because just 41 points are calculated. See Figure 6.

Number of Points Calculated, ⌘-N

You can change the number of points calculated for a graph to values between 2 and 1000 by using ⌘-N.

Example 18 (continued): Continue the present example by changing the number of points calculated for the graph. To change the number of points calculated, use the ⌘-N, Number of Points command. For example,

press ⌘-N and change the number of points evaluated to 20.
Press ⌘-E, Return to erase, and
press ⌘-G, Return to replot.

Plotting takes less time, but the graph is not smooth. Now, erase again and try 100 points. Notice that it takes longer to plot but produces a smoother graph.

Unconnected Points, ⌘-U

You can see an unconnected graph by using the "Unconnect or Connect Points" command.

Example 18 (continued): To see the calculated points plotted but not connected, continue the present example:

press ⌘-U for the "Unconnect or Connect Points" command,
select UNCON,
Press ⌘-E, Return to erase, and
press ⌘-G, Return to replot.

Notice the different style of graph produced by unconnected points. Then,

Press ⌘-U again,
select CON to connect points. Then,
press ⌘-G, Return to replot the graph.
This time, the points are connected.

Plotting graphs with unconnected points is useful for graphs with discontinuities. Here is another example. In this example, we will begin to abbreviate instructions by not indicating every Return that you must press.

Example 19: First press ⌘-E and select BOTH to erase. Then,

press G and define G(X) = INT(X),
press ⌘-U and select UNCON,
press ⌘-N, enter 200, and
press ⌘-G and select G to graph G(X).

The 200 points specified here will fill in the horizontal segments of the graph of this step function (see Figure 7). And, since points are not connected, the jump discontinuities are displayed correctly. To see an incorrect graph, press ₵-U and select CON. Then, press ₵-G and graph G again.

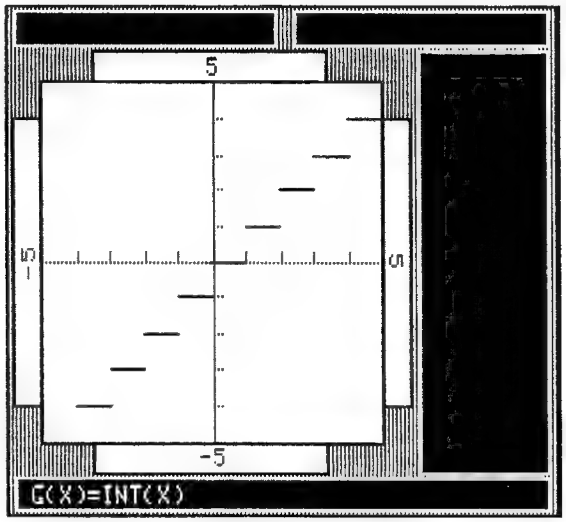

Figure 7.

Changing the Coordinate Window, ₵-W

There are three ways to change the coordinate window on which graphs are plotted. One way is to assign "bounds" to each side of the coordinate window. Another is to "frame" a section of the current coordinate window and zoom in on that section. A third is to "zoom" during the ₵-P, Project/Plot routine.

Here is an example using BOUNDS to change the coordinate window.

Example 20: First, press ₵-R to restart, so that you have the default coordinate window displayed. Then,

press ₵-W for WINDOW COORDS, select BOUNDS,

enter –10 for Left Side,

10 for Right Side,

–10 for Bottom, and

10 for Top.

As each entry is made, its current value is printed in the appropriate position on the boundary of the coordinate window. When you enter the Top value, the old coordinate window is erased (together with any graphs on it), and the new coordinate window is presented. Also, the plotting interval (see ₵-I in Appendix 6) is set automatically to [–10,10].

Then the prompt for X-Tick appears. Notice that the distance between tick marks is currently one unit on each axis. If you are satisfied with the tick marks on the coordinate axes, simply press Esc to return to the ? prompt. To change the distance between the tick marks, enter the new distance between X-axis tick marks at the X-Tick prompt.

Then enter the distance between the Y-axis tick marks at the Y-Tick prompt. For example, entering 2 for both X-Tick and Y-Tick in the current example produces half as many tick marks on each axis. They are now two units apart.

If an axis is outside a selected coordinate window, it is not plotted. Also, if tick marks are too close together, they do not appear, even though the axis is inside the coordinate window. Entering a zero or negative tick distance produces no tick marks.

If Esc is pressed before all four sides have been entered, the old coordinate window and plotting interval are retained, and graphs are not erased. You can use Esc to start over. The right side value must be greater than the left side, and the top greater than the bottom. Otherwise, the entry is rejected, and the prompt reappears for another entry or Esc.

You may use BASIC expressions to define the values entered for coordinate bounds and tick marks.

Example 21: To plot the graph of F(X) = A * SIN(B * X + C), it might be convenient to use –2 * π, 2 * π, –4, and 4 for the respective bounds. Try it. (Remember that pressing % prints π on the Graphics display.) Also, use π/2 for the X-Tick distance and 1 for the Y-Tick distance. Then define A = 3, B = 1, and F(X) = A * SIN(B * X + C). Finally, press ♂-G to plot the graph of F.

If functions or parameters other than π are used to define a coordinate window, a decimal approximation to the current value of the expression is used to label the bounds of the coordinate window. Fractions, however, are used as expressions. For example, the respective bounds may be labeled –3/4, 3/4, 1 – 3/2, and 1 + 3/2. On the other hand, if U – W, U + W, V – W, V + W are used, the bounds appear as decimal approximations of the current values of these expressions. These current values must be valid bounds entries, or they are rejected. For example, the current value of W in the previous expressions must be positive, so U – W < U + W and V – W < V + W.

Example 22: Define U = 1, V = 2 and W = 10. Press ♂-W and select BOUNDS. Then, enter

U – W for Left Side,
U + W for Right Side,
V – W for Bottom,
V + W for Top,
W/5 for X-Tick, and
W/5 for Y-Tick.

This produces a coordinate window 2 * W units wide, 2 * W units high, and with distance W/5 between tick marks. The center (not the origin) is at the point (U,V).
Now, define U = 3, V = –2, W = 5, and change the coordinates again. Notice that the expressions need not be retyped. For example, just press Return when U – W appears for Left Side.

The technique in the last example is useful for centering the window about the point (U,V). The next example shows you another way to position a coordinate window.

Framing a Graph

The values for the left side, right side, bottom, and top of the coordinate window may also be assigned by framing a section of the current coordinate window with movable borders. You can zoom in on a section and replot graphs there. Here is an example:

Example 23: To begin, choose

Ȯ-R to restart,
G to define G(X) = SIN(π * X), and
Ȯ-G to graph G.

Now, suppose that you want to focus on just one period of the graph of G—say, for X between –1 and 1. To do this, press Ȯ-W to change WINDOW COORDS, and select FRAME. When the prompt USE ARROWS TO MOVE LEFT SIDE appears, press → to move the left border. Each time → is pressed, the border moves the distance Delta-X to the right, and its location, X, is printed as the left coordinate bound. Pressing ← moves the border to the left. Holding down the ← or → cursor keys moves borders more rapidly.

You may change Delta-X using Ȯ-X, just as you did with Ȯ-P for Project/Plot. Similarly, you may change X by pressing X and changing the value for X. This helps locate the left side exactly where you want it and, in particular, allows for rounding values of X. Frequently, depending on arithmetic approximations, 9-digit decimal numbers occur for X. You can round these for neater displays or for values which are easier to understand.

When the left side bound is at –1, press Return to enter that value. Then you are prompted to USE ARROWS TO MOVE RIGHT SIDE. Press ← to move the right side left to 1. Of course, Ȯ-X may be used here to change Delta-X, and you may assign or change the value of X by pressing X. When you get the right side where you want it, press Return to enter the current X-value for Right Side.

Then press ↑ to move the Bottom up to –1.25. For Y-values, press Ȯ-Y to change the increment, Delta-Y; press Y to assign a bound location directly to Y. When the bound is where you want it, press Return to enter the current value for Bottom.

Finally, use ↓ to move the Top down to 1.25. When you press Return to enter this Y-value for Top, the coordinate window is redrawn, and you are prompted to enter the X-Tick distance. Enter 1/4. Also enter 1/4 for the Y-Tick distance, then press Ȯ-G to regraph G, producing just one period of the graph as desired.

Remember that the values of X, Y, Delta-X, and Delta-Y are changed globally (everywhere) whenever they are changed, not just in the Ȯ-W, Window Coordinates routine. And, you can use BASIC expressions involving functions and parameters to assign appropriate values. In the Ȯ-W, frame routine, X and Y must be assigned values inside the current coordinate window, and you cannot assign "Right Side ≤ Left Side" or "Top ≤ Bottom." Also, Delta-X and Delta-Y are always positive.

To return to the default window coordinates, you can press Ȯ-R to restart (this will set all functions and parameters to zero!), or you can press Ȯ-W to select any window coordinates you want.

Changing Graph Colors, ⌘-C

There are several combinations of colors and linestyles available for plotting graphs.

Example 24: First, press ⌘-R to restart. Define F as F(X) = A * SIN(B * X + C) and A = 3, B = 1.5, and C = 0. For the sake of brevity, the Return key is not indicated in the sequence of keys below.

Now,
press ⌘-G to graph F in white,
press ⌘-C for the Color of Graph command, and
select VLT for violet. Now,
press ⌘-G to plot the graph of F in violet.

Then try GRN for green and BLK for black. Observe that black erases the graph. Finally, press ⌘-C to select HRS for high resolution. The graph is multicolored but not as thick as the previous graphs.

Combinations of colors are also possible . For example, press

⌘-E to erase,
⌘-N to select 171 points,
⌘-C to select white,
⌘-U to select unconnected,
⌘-G to graph F,
⌘-N to select 86 points,
⌘-C to select green, and
⌘-G to graph F.

The result is an unconnected graph of F with green and white points. Now, press
⌘-C to select violet and
⌘-G to graph F.

The green points are changed to violet.

You might wonder why we chose 171 or 86 points in the last example. The reason is that these values produce a fairly smooth graph. The coordinate window on *Graphics Calculator* is 170 pixels wide. (A pixel is an addressable point of light on the monitor screen.) The smoothest graphs occur when the number of points used is 43, 86, 171, 341, or 681, because these values minimize rounding on the X-coordinates of the points plotted.

Clipping to the Coordinate Window

The next example shows what happens when calculated points are outside the coordinate window.

Example 25: This example depends on the default setup, so to begin,

press ⌘-R to use the restart command,
define F(X) = A * SIN(B * X + C), A = 6, B = .5, and C = 0,
press ⌘-U and choose UNCON for unconnected plotting; and
press ⌘-G to plot the graph of F.

If a calculated point is outside the coordinate window, it is not plotted. A small arrow inside the coordinate window points to it, and a sound alerts you of the event. Now,

press ⌘-U and choose CON for a connected graph, and

press ⌘-G to graph F.

Observe that the segments connecting the calculated points are "clipped" at the edge of the coordinate window. That is, only that part of a segment inside the coordinate window is plotted.

Plotting Tables of Values, ⌘-V

You can easily display a table of your own values on the Graphics display and plot individual points corresponding to that table.

Example 26: Here, we plot individually selected points before graphing.

Press ⌘-R to restart,

press H and define H(X) = X * X – 4.5,

press ⌘-V for the Values Assigned command,

select H to graph H(X), and

enter X = –3 at the upper left prompt.

When you press Return to enter –3, the value of H(–3), Y = 4.5, is displayed in the upper right display, and both the X- and Y-values are recorded in the right-hand table. The corresponding point is plotted on the coordinate window. Then you are prompted to enter another X-value; enter –2, –1, 1, 2, and 3 consecutively for X. Each time you press Return, the corresponding Y-value is calculated using Y = H(X) and is recorded along with X in the right-hand table. Also, each point is plotted.

Now, press Esc to exit the ⌘-V, Values Assigned routine, and press ⌘-G to graph H(X).

You can also emphasize points on a graph that is already plotted by using ⌘-V. Plot the graph in violet or green, then use ⌘-V to plot white points on the graph. White is always used for the individual points, regardless of the color chosen with ⌘-C. Try repeating the last example by plotting the graph of H(X) in violet. Then use ⌘-V to enter the values –3, –2, –1, 1, 2, and 3 for X.

⌘-V also works in conjunction with Project/Plot (see *Projecting Values from Graphs, ⌘-P*).

Plotting Polygons

In the previous example, you might have noticed the last selection, POLY, on the ⌘-V, Values Assigned menu. This selection allows you to enter both the X- and Y-coordinates of points. If the ⌘-U option is set on CON (connected), the consecutive points are joined by line segments in the current ⌘-C color. This produces a polygon or polygonal line. If ⌘-U is set on UNCON (unconnected), the points are plotted but not joined.

Example 27: Press ⌘-E at the **?** prompt and select BOTH to erase. Then press ⌘-V and select POLY. Now enter:

X = 4 and Y = 4,
X = –4 and Y = 4,
X = –4 and Y = –4,
X = 4 and Y = –4, and
X = 4 and Y = 4.

As the points are joined by line segments, a square is plotted. You can draw any polygon by entering the consecutive vertices.

Plotting on Intervals and Function Domains, ⌘-I

There are times when it is useful to restrict the interval on which a graph is plotted. Here is an example.

Example 28: To return to default status,

Press ⌘-R to restart.
Press G and define G(X) = SQR(X), and
press ⌘-G to plot the graph of G.

Notice that for X < 0, G(X) is not defined; as attempts are made to calculate G(X), the computer speaker "clicks" to alert you that an undefined calculation was detected. Thus, the graph of G is plotted only on the intersection of its domain with the segment of the X-axis in the coordinate window. If two consecutive successful calculations occur, the corresponding points are connected (when ⌘-U is set to the CON option).

To avoid large numbers of unsuccessful calculations, you can restrict the interval from which the X-values for plotting are taken. Here is how:

Press ⌘-I for the Interval Choice command,
enter 0 for X-INT LEFT (left endpoint),
enter 4 for X-INT RIGHT (right endpoint),
press ⌘-E to erase graphs, and
press ⌘-G to replot the graph of G.

Note that the graph is plotted only on the interval [0,4]. As usual, the number of points plotted in the interval is controlled by ⌘-N. If you have not pressed ⌘-N since restarting, 41 points were plotted on [0,4].

At times, depending on computer rounding, the right endpoint of a plotting interval may be rejected, even though it is theoretically in the domain of the function.

Example 29: We assume that the Interval Choice is still [0,4] as in the last example.

Press ⌘-E to erase,
press H and define H(X) = SQR(16 – X * X),
press ⌘-N to set the number of points to 50, and
press ⌘-G to plot the graph of H on the interval [0,4].

There is a "click" as the right endpoint, 4, is rejected, and the graph of the circular segment does not reach the X-axis as it should. The computer has calculated an

x-value slightly greater than 4 for the last point and found it outside the domain of H. Note that H has domain [–4, 4].

Press ♂-N to change the number of points to 100, then replot the graph. This time, the graph reaches the X-axis.

The plotting interval chosen with ♂-I must be a subinterval of the segment of the X-axis in the coordinate window. For example, when the right endpoint of the X-axis is 5, the right endpoint of the plotting interval must be less than or equal to 5. It must also be greater than or equal to the left endpoint. Entries that do not satisfy these conditions are rejected, and the prompt to enter reappears.

If the coordinate window is changed using ♂-W, the plotting interval changes automatically to coincide with the segment of the X-axis in the coordinate window.

The ♂-I, Interval Choice command is also useful for graphing functions with vertical asymptotes, as you will see in the next example.

Example 30: First,

press ♂-R to restart with the default setup,

press F to define F(X) = X/(X * X – 4), and

press ♂-G to plot the graph.

The result is not very good. This function has vertical asymptotes at X = –2 and X = 2. Only 41 points are plotted, so no points are plotted very near the asymptotes. To remedy this, press ♂-N to increase the number of points to 161, and press ♂-G to replot.

This method of increasing the number of points does not always work. For example,

press ♂-N to change the number of points to 100,

press ♂-E to erase the previous graphs, and

press ♂-G to replot the graph of F.

This time, points on the opposite sides of the asymptotes have been connected, producing vertical line segments at the asymptotes. This does not happen with 41 or 161 equally spaced points in [–5, 5], because X = –2 and X = 2 are tried as values in F(X) and rejected. For a further explanation, see the sections on "Error Propagation" and "Avoiding Error Propagation" in Appendix 10.

To obtain a graph without the vertical segments you saw in the last example, a method that always works is to plot the branches of the graph between asymptotes separately. Use ♂-I to restrict the plotting interval appropriately. We continue the last example . . .

Example 30 (cont'd): The function, F(X), is defined as in the beginning of this example. To continue, define the small increment, B = 1E–4, to save some typing. Then press:

♂-N to set the number of points to 26,

♂-E to erase the old graph,

♂-I to set up the interval [–5, –2 – B],

♂-G to plot the left branch,

♂-I to set the interval to [–2 + B, 2 – B],

♂-G to plot the middle branch,

♂-I to set the interval to [2 + B, 5], and

♂-G to plot the right branch.

Later, another way to plot graphs with asymptotes is demonstrated, using ⌘-P.

The branches of piecewise-defined functions may be graphed in much the same way as the branches in the last example.

Example 31: The function:

$$f(x) = \begin{cases} x + 3 \text{ for } -5 \le x < -2 \\ x^2 - 4 \text{ for } -2 \le x \le 5 \end{cases}$$

may be graphed two different ways. One way is to graph two functions on separate intervals. First, press ⌘-R to restart with the default setup. Then press:

G to define G(X) = X + 3,
⌘-I to select the interval [–5, –2],
⌘-G to graph G(X), the left branch of f(x),
H to define H(X) = X^2 – 4,
⌘-I to select the interval [–2, 5], and
⌘-G to graph H(X), the right branch of f(x).

A second way to obtain the graph is to use logical expressions to define a single function, F. First, press ⌘-R to restart. Then, press

F to define F(X) = (X + 3) * (X < –2) + (X^2 – 4) * (–2 < = X),
⌘-U to select UNCON,
⌘-N to select 300 points, and
⌘-G to graph F.

Notice that the graph would be incorrectly connected at the point X = –2 if you selected CON rather than UNCON for the ⌘-U option.

The next section, *Projecting Values from Graphs, ⌘-P,* illustrates a third way to graph piecewise-defined functions.

Projecting Values from Graphs, ♂-P

Using the ♂-P, Project/Plot command is another way to plot graphs. At the same time that points are plotted, their X- and Y-coordinates are available at the top of the graphics display. These X- and Y-values may be assigned to parameters for use in calculations on the Calculator display. As projections move, small arrows indicate their position on the X-and Y-axes.

Points may be plotted as connected or unconnected, and in several colors, just as with the ♂-G, Graph Function command. Different sections of graphs may be plotted separately and in different colors. The number of points plotted may be changed using the ♂-X, X-increment command (rather than the ♂-N, Number of Points command, which affects only the ♂-G command, not the ♂-A, Array command or ♂-P, Project/Plot).

Projections and traces help visualize the correspondence between the domain and range of a function and also show how this correspondence affects the shape of the graph of the function.

Example 32: First, press ♂-R to restart with the default setup. Then,

press F to define F(X) = 3 * SIN(X),

press ♂-P and select ARROWS from the first menu, and

select F from the next menu.

When Return is pressed, the values X = 0 and Y = 0 are printed at the top of the Graphics display, and two small projection arrows appear at the origin. To plot the graph of F, press one of the cursor keys, ← or →. The projection arrows move and new values appear at the top of the display. Also, a large white point indicates the last point plotted.

Each time the → cursor key is pressed, the X-value is incremented by 0.25. If ← is pressed, the increment is –0.25. If the ♂-U option is set on CON, the new point is connected to the previous point as it is plotted.

To plot the graph more quickly, hold down one of the cursor keys. The coordinates at the top of the screen are not changed until the keys are released. This allows the X-projection to move left and right more rapidly as ← or → is held down.

As the X-projection is moved across the entire X-axis, the complete graph is plotted. Observe how the Y-projection oscillates periodically as X moves from right to left.

To leave the Project/Plot routine, press Esc, and the ? prompt returns to the lower right of the screen.

Controlling Projected Values, ♂-X and X

Delta-X, the X-increment used by the Project/Plot routine, may be assigned any positive value that is valid in BASIC. This assignment may be made at the ? prompt or during the Project/Plot routine. Simply press ♂-X for the X-increment command and enter your expression for Delta-X. You may use functions and parameters in an expression to define Delta-X (just as when assigning values to parameters), but the

current value of the expression must be positive. Otherwise, it is rejected and the input prompt returns.

The expression you enter for Delta-X becomes the current expression, and its value is the current value for Delta-X, just as with parameters. The next time you press ♂-X, the current expression is presented for modification. (See the chapter below entitled *♂-S, Checking the Setup,* and also see Appendix 6).

The value of the variable, X, may also be assigned at the ? prompt or during the Project/Plot routine. Just press X, and enter an expression for the desired value of X. The only restriction on the value of X is that it must be in the current interval as defined with the ♂-I, Interval Choice command. If you have not defined an interval, the default interval is the current coordinate window's X-axis. If X is not in the current interval, the value of X is changed automatically when Project/Plot starts, and you are notified of this event.

Since X is a variable and not a parameter, it does not have a current expression different from its current value. The current value of X, rather than the last expression used, is always presented for modification.

The values of Delta-X and X may be changed at the ? prompt, before starting the Project/Plot routine, or while the Project/Plot routine is in progress, as in the following example:

> *Example 33:* Begin at the **?** prompt, and
>
> press F and define F(X) = 3 * SIN(X),
>
> press X and enter 1,
>
> press ♂-P and select ARROWS, and
>
> select F.

When you press Return to enter the function selection, the projections start with X = 1, just as you entered it. Now, press X and enter 2. The projections immediately move to X = 2. You can move the X-projection to any X- value in the current interval.

Now, press

> Esc to end the projections routine,
>
> ♂-E to erase the graph, and
>
> ♂-P to project F again.

Observe that the X-projection starts exactly where you left it, at X = 2. The X-projection always starts at the current value of X if it is in the current interval.

Before using the cursor keys to move the projections,

> press ♂-X to change Delta-X, and
>
> enter 1/16 as the new expression.

Now, each time you press a cursor key, the X-projection is incremented by 1/16 or –1/16. If you hold down a cursor key, the graph is plotted more slowly, because more values are calculated and more points are plotted.

Now, press

Esc to leave the projections routine,
-E to erase the graph,
-U to select UNCON,
-X to enter 1/4 for Delta-X,
-P to project F again, and
and to plot the graph.

Now, individual points are plotted but not connected. The points plotted depend on the starting value, X, and on the increment, Delta-X, selected. For example, use the cursor arrows to move X all the way to the right, then back to the extreme left, so that all points are plotted with Delta-X = 1/4. Then, press

Esc to leave the projection routine,
-C to change the color to green,
X to define X = X + 1/8,
-P to project F again, and
← and → to plot green points.

Since the definition, X = X + 1/8, moves the starting value of X 1/8 unit to the right (exactly half the increment, Delta-X = 1/4), the green points are between the previously projected white points. Project all the green points possible. Now, press -X to change to Delta-X = 1/8 . Then, when you press ← and →, twice as many points are plotted, and all of the white points are changed to green.

To end this example, press Esc to leave the projection routine.

Assigning Projected Values to Tables, -V

While you are projecting the graph of a function, the current X- and Y-values may be displayed in the right-hand graphics table. Just press -V whenever you have paused in the Project/Plot routine. The values of X and Y that appear at the top of the display are reproduced in the right-hand table. If the number of digits is too large for the space allotted, the values are truncated on the right, and a question mark (?) appears as the last character.

Notice that -V is used as a second-level command here. As a first-level command (when the **?** prompt is present), it has a different effect.

Here is an example.

Example 34: Suppose you want to make a table of values for F(X) = X * X − 4, with equally spaced values of X, and at the same time plot the corresponding points. Just press

-R to restart. Then,
F to define F(X) = X * X − 4,
-U and select UNCON,
-P and select ARROWS and F, and
-X and enter .5 for Delta-X.

The current value of X should be X = 0. If not, press X and assign X = 0. To record the coordinates of this starting point in the table, press ⌘-V . Then alternately press → and ⌘-V five more times to plot points and fill up the table (see Figure 8). The next time you press ⌘-V, it will replace the top entry.

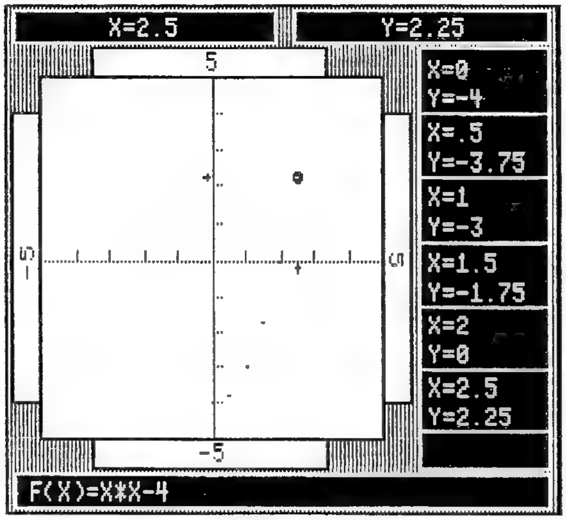

Figure 8.

You may also choose X explicitly by pressing X and entering the next value of X. For example, press X, enter X = –2, and press ⌘-V to record the resulting intercept in the table.

Press Esc to end the Project/Plot routine.

Assigning Projected Values to Parameters

During the Project/Plot routine, you may assign the projected X- and Y-values as the current values of parameters. To see how, we continue from the last example.

Example 35: We assume that you have just finished the previous example. To assign a projected X- or Y-value to a parameter, just press the corresponding key during the Project/Plot routine.

First, press ⌘-P and select F(X) to start the Project/Plot routine. Then press I to assign the current value of X to I. In the table on the right side of the Graphics display, you see the choice I = [X] Y. Press the cursor keys, ← and →, to choose between X and Y. This time, choose X. Press Return to complete the assignment of the current value of X to I. The value of I appears in the table. This assignment changes only the current value of I, not the current expression.

To assign the current value of Y to the parameter, J,

press J,

use ← and → to select J = X [Y], and

press Return to complete the assignment.

The value of J is recorded in the table. Values displayed in the table are truncated if they do not fit. The last character of a truncated value is a question mark (?). This is for

display purposes only . The value stored internally is not truncated. If an assigned value is used later in a calculation or larger display area, the full precision is used.

When you have plotted the desired points and recorded the desired values in the table, press Esc to end the Project/Plot routine. You can start a new table by pressing ⌘-E to erase values or both graphs and values. (Choose VALUES or BOTH.)

Tracing the Mapping of Domain to Range, ⌘-M

Previous examples illustrate how the Project/Plot command, ⌘-P, is used to plot the graph of a function while observing or storing the coordinate values in various displays. It is also possible to trace the mapping of these domain and range values graphically on the X- and Y-axes respectively. This helps you to visualize a function as a mapping. The command used is ⌘-M for Mapping Trace. This second-level command turns the Mapping Trace feature on and off; it works only while the Project/Plot feature is being used. At the **?** prompt, the first-level command ⌘-M is used to change modes.

Example 36: To begin, press ⌘-R to restart with the default setup. Then, press

F to define F(X) = (X^2)/4 – 4,
X to enter 1, and
⌘-P to select ARROWS and F.

Now the white projection point is displayed with coordinates X = 1 and Y = –3.75. Before using the left and right cursor keys to project other values, press ⌘-M to activate the Mapping Trace feature. Observe that the small projection arrows move away from the axes slightly and the coordinates flash once. You are ready to trace some values.

Press → once; observe that X is incremented just as before (Delta-X = .25), and a segment of the graph is plotted. But small segments are also traced on the X- and Y-axes. Press → again, and the process is repeated. Press → twice more, and the coordinate display shows X = 2 and Y = –3. The interval [1, 2] is traced on the X-axis, while the interval [–3.75, –3] is traced on the Y-axis. The movement of the projection arrows gives a good indication of how F maps the interval [1, 2] onto the interval [–3.75, –3]. Note that F is increasing and one-to-one on [1, 2].

Incrementing X four more times sets the coordinates on X = 3 and Y = –1.75. The traces now show that F maps [1, 3] onto [–3.75, –1.75].

Now we repeat the process. But this time we save the endpoint values to use later in the calculation of a slope. To repeat the process, use ← to move X back to 1 (or just press X and enter 1).

The projected values may be assigned to the parameters A, B, C, etc. while the Mapping Trace feature is operating. With X = 1,

press A, and observe the right side table,
select A = [X]Y using ← and →,
press Return to assign the value of X to A,
press B,
select B = X[Y] using ← and →, and
press Return to assign the value of Y to B.

The last Return returns you to Project/Plot to project and trace more values. So,

use ← and → to move X to X = 3,
press C,
select C = [X]Y,
press Return,
press D,
select D = X[Y],
press Return, and
press Esc to leave the projections routine.

In this way, coordinates of points are saved as current values of parameters for later calculations. For example, the parameter definition M = (D – B)/(C – A) assigns to M the slope of the line segment from the point (A, B) to the point (C, D). The traces on the axes show the rise and run geometrically.

Mapping Traces may also be made for unconnected graphs. This produces a picture of the mapping of a *regular partition* of the domain (a set of equally spaced points in the domain). To see this, we continue the last example.

Example 37: To begin, press

Ċ-E to erase BOTH graphs and tables,
Ċ-U to select UNCON,
X to enter X = 0,
Ċ-X to enter .3 for Delta-X,
Ċ-P to start Project/Plot for F, and
Ċ-M to start the Mapping Trace.

Now hold down → to trace a partition of equally spaced points (Delta-X = .3) on the X-axis. Observe the variation in the distance between the corresponding range points. If you watch closely, you'll see that as the X-projection moves with constant velocity, the velocity of the Y-projection varies depending on the location of X. To emphasize the varying velocity of Y, press Ċ-X to change Delta-X to .1. Then press Ċ-M to shut off the Mapping Trace, and hold down the ← or → keys to move X left or right.

Sketching Graph Details with Project/Plot

The Project/Plot command is also useful for "sketching" sections of graphs in more detail or in more than one color. Near vertical asymptotes or domain endpoints, the points plotted by Ċ-G, Graph Function may sometimes be inadequate. But Ċ-P, Project/Plot is more versatile.

Example 38: To begin, press

Ċ-R to restart,
F to define F(X) = SQR(49/9 – X * X), and
Ċ-G to graph F.

The graph should be a semicircle of radius 7/3. Notice that the graph is incomplete because the endpoints, –7/3 and 7/3, were not among the equally spaced X-values used for plotting. For graphs of this type, it is often better to use Ċ-P, Project/Plot to plot the graph. Press

⌘-E to erase the graph,
X to enter X = –7/3,
⌘-P to select ARROWS and F, and
→ to plot the graph.

Again, the right end of the graph may not meet the X-axis, but this is easy to fix. Just press X, enter 7/3, and press ← to fill in the missing segment.

By choosing the starting value for X and the value of Delta-X carefully, important details of graphs that plotting with ⌘-G, Graph Function may omit, are easily "sketched in." The following graph with asymptotes is an example.

Example 39: First, press ⌘-E at the **?** prompt to erase the graphs. Then, press

F to define F(X) = X/(X * X – 9),
B to define B = 1E –4,
⌘-X to enter .1 for Delta-X, and
⌘-P to select ARROWS and F.

Since F has vertical asymptotes at X = –3 and X = 3, you want the X-projection slightly left of –3 before plotting. So, press

X to enter X = –3 – B,
← to plot the left branch of the graph,
X to enter X = –3 + B,
→ to plot the middle branch to X = 0 approximately,
X to enter X = 3 – B,
← to finish the middle branch,
X to enter X = 3 + B,
→ to finish the right branch of the graph, and
Esc to exit the Project/Plot routine.

Defining B = 1E –4 saves some typing in this example, and defining Delta-X = .1 makes the increment small enough for reasonable detail. This also makes it easier to control the plotting near asymptotes. If Delta-X is too large, you can jump past an asymptote and get an incorrect graph.

It is possible to modify the method of the last example and not use B. If you don't know exactly where the asymptotes are, plot an unconnected graph first to get a general idea. Then use ⌘-P, Project/Plot to plot the connected graph. Each time you approach an asymptote, press X and change the current value of X to skip past the asymptote. With a little care, you can sketch the graph.

Projecting with Arrows or Lines

The "lines" option from the ⌘-P, Project/Plot menu produces a version of the Project/Plot routine slightly different from the one produced by the "arrows" option we have been demonstrating. In place of the large white point on the graph and the two projection arrows, you see a vertical projection line from the graph to the X-axis and a horizontal projection line from the graph to the Y-axis. For some purposes (estimating intercepts of graphs, for example), these projecting lines are easier to use than the point-arrow combination. The large point and arrows may obscure intercepts. On the other hand, if a graph is nearly vertical or nearly horizontal, the arrows work better,

because the lines erase parts of the graph. You may want to repeat a few of the previous examples (or try some of your own) with the Lines option. The next section on "zooming" also shows the use of lines.

Zooming on Important Points, ⌘-Z

During the Project/Plot routine, it is possible to zoom in on zeros of functions, maximum/minimum points, or intersection points of graphs, while projecting increasingly accurate approximations of their coordinates. You can quickly approximate zeros of functions, roots of equations, maximum or minimum values and intersection points of graphs to eight or nine digits by combining graphic and numeric techniques.

Zooming on Extreme Points

Here is an example of approximating the coordinates of a local extreme point:

Example 40: First press ⌘-R to restart. Then, define F(X) = 3 * SIN(X), and press ⌘-G to plot the graph.

Suppose that you want to find the coordinates of the maximum point located between X = 1 and X = 2. (Of course, it is at X = π/2, but suppose that you did not know this.) To begin, start the Project/Plot routine for F. That is, press ⌘-P and select ARROWS and F.

To approximate the coordinates of the maximum point between X = 1 and X = 2,

press ← and → to adjust X to X = 1.5,
press ⌘-Z for the Zoom command,
enter 10 for X-MAGNIF and press Return, and
enter 100 for Y-MAGNIF and press Return.

Immediately, a new coordinate window is displayed, with the projection point at the center you specified. The X-scale is magnified by the factor 10, while the Y-scale is magnified by 100. That is, the width of the displayed coordinate window is just 1/10 the previous width, and the height is 1/100 the previous height. Also, the increment, Delta-X, is automatically multiplied by 1/10, so it is better suited to the new coordinate window. (You may reassign your own Delta-X, by pressing ⌘-X, if you wish.)

Each time the projection arrows and point reappear, you may repeat the process by pointing to a new center of magnification. Use ← and → to plot a segment of the graph and search for the maximum point.

Watch the Y-coordinate. Since you are searching for a maximum point, try to adjust X so that the displayed value of Y is as large as possible. Then, press ⌘-Z to zoom on the new center. When prompted for X-MAGNIF just press Return to use 10 again. Also, enter 100 again for Y-MAGNIF.

Again, use ← and → to plot an appropriate segment of the graph and choose the new center, so that the Y-coordinate is as large as possible. Then press ⌘-Z and repeat the process.

Soon, the largest Y-coordinate displayed is 3, and the corresponding X-coordinate is approximately 1.5708. The exact maximum point, (π/2, 3), is approximated quite well. To end the ⌘-P, Project/Plot routine, press Esc and the **?** prompt returns.

Zooming on Function Zeros

Zooming also works well to approximate zeros of functions. For zeros, using the LINES option to project the center of magnification is often preferred over using the arrows option.

Example 41: First, press ♂-R to restart with the default setup. Then define F(X) = 3 * COS(X). To approximate the zero of F(X) at X = π/2, begin by pressing ♂-P to select LINES and F. Now use ← and → to plot the graph of F. When the graph is complete, adjust X to near X = 1.5, so that Y is near zero. Then the horizontal projection line is near the X-axis.

You can get Y closer to zero, and the horizontal projection line closer to the X-axis, by pressing ♂-X to change Delta-X to .025. Then adjust X again. You should be able to make the horizontal projection line coincide with the X-axis (making it is green, rather than white, on a color monitor.) Now press ♂-Z to zoom. Enter 10 for the X-MAGNIF and 10 for Y-MAGNIF. The new coordinate window is displayed and the projections return. Delta-X is updated automatically, as in the last example.

When the projection lines return, repeat the process. Plot a segment of the graph near the zero and adjust X so that Y is as near to zero as possible. The horizontal projection line (the Y-projection) is as close to the X-axis as possible at this time. It may even be on the X-axis (if it is green). Then press ♂-Z to zoom; enter 10 for X-MAGNIF and 10 for Y-MAGNIF.

Each time the process is repeated, an improved approximation to the zero is obtained. Compare your approximations to X = 1.57079633 as you proceed.

To save the value of a given approximation, just assign it to a parameter. To assign a value to A, press A, use ← or → to select X or Y in the right-hand table, and press Return. The current value of X or Y is assigned to A and is displayed in the table (perhaps in truncated form). The current expression for A is not changed. You may also press ♂-V, to display values in the table without assigning them to a parameter. See Examples 26 and 35 for details.

After 7 or 8 iterations, you may obtain Y = 0 exactly, but this is not always possible. One reason is the inexact arithmetic computers use (see Appendix 10, *Computer Arithmetic Errors*). In this example, since you know that X = π/2 is the zero, you can simply assign X = π/2 (press X and enter π/2 using the **P** or **%** key for π) and obtain Y = 0.

To exit the ♂-P, Project/Plot routine, press Esc and the **?** prompt returns.

Restarting after You Zoom

Each time you have finished zooming, you can return to the default setup using ♂-R for Restart. Remember that restart not only returns you to the default coordinate window but also sets all function and parameter definitions to zero. To retain function and parameter definitions, you can use ♂-W, and select BOUNDS to reset the coordinate window. Then use ♂-X to change Delta-X back to .25. (See *Changing the Coordinate Window*, ♂-W.)

It is also possible, as you will see later, to save and reload various other default setups automatically, without the need to re-enter function definitions or other expressions. (See *Loading and Saving Setups, ⌘-S.*)

More Zooming Details

To zoom on a point that is on a nearly horizontal part of a graph, you may need to use a value of 100, 1000 or more for Y-MAGNIF, as in Example 40. If the graph is nearly vertical, use these larger values for X-MAGNIF.

If you continue zooming until your next magnification factors would (due to rounding) produce coinciding left sides and right sides or top and bottom for the coordinate window, the Project/Plot routine halts automatically, and the ? prompt returns. If the graphs and projected values become unreliable before the automatic halt (usually when coordinate bounds agree to 8 significant digits or more), press Esc to halt.

It is not a good idea to use extremely large magnification factors. It is better to use smaller magnification factors several times, adjusting the center of magnification each time. An extremely large magnification factor could result in the Project/Plot routine halting as described in the last paragraph. Or, zooming in too fast could put the point you are trying to approximate completely outside the new coordinate window. This is because the magnification center (the currently projected point) is often only an approximation of the actual point on which you are trying to zoom in. If you zoom in too far on this approximate point, the point you want will be outside the coordinate window. Of course, if you know the exact coordinates of the point you want to zoom on, project these before you begin to zoom, and you will avoid the problem just described.

If you accidentally zoom in too far, don't move the projected center. Press ⌘-Z to keep the same center, but use the reciprocals of your last magnification factors to zoom out rather than in. (You can zoom out by using magnification factors between between 0 and 1).

It is also possible that the X-increment, Delta-X, becomes so small that it is negligible when added to X. Then the projections no longer move, because X does not change. (They just flash in one place.) Use Esc to exit or ⌘-X to adjust Delta-X. On the other hand, if you press ⌘-X and make Delta-X larger than the width of the coordinate window, the projections again cannot move. This time, the X- and Y-values flash, the projections do not. Press ⌘-X to adjust Delta-X.

We repeat that the ⌘-Z, Zoom routine does not cause the projection routine to exit automatically until the computer detects "Left Side = Right Side" or "Top = Bottom." Before this happens, the graphs or function values could become unreliable. So, you may want to press Esc to exit when your approximation is close enough for your needs.

The current values of X and Y satisfy "Left Side ≤ X ≤ Right Side" and "Bottom ≤ Y ≤ Top."You can use this information to estimate the difference between X and Y and the exact values they approximate.

Zooming on Intersection Points of Graphs

You can change from one function to another while you are projecting values and plotting graphs. In the next example, we introduce this feature of the ♂-P, Project/Plot routine by zooming to approximate the intersection point of two graphs. In this example, ♂-F is used as a second-level command to switch between graphs of different functions while projecting. Also, ♂-Z is used to zoom on the intersection point.

Example 42: The objective is to approximate an intersection point of the graphs of the functions F(X) = EXP(X) – 2 and G(X) = X + 1.

To begin:

Press ♂-R to restart with the default setup.

Define F(X) and G(X) as indicated.

Press ♂-G to GRAPH F first, then G.

Press ♂-P, select ARROWS and G, and project the intersection point on the right.

Press ♂-Z to zoom. Enter 10 for both X-MAGNIF and Y-MAGNIF.

When the large white projection point returns, you must replot sections of each graph to obtain the intersection point again. The current function is still G, the function last projected, so press:

← and → to plot the graph of G,

♂-F, select F, and press Return,

← and → to plot the graph of F,

← and → to project the intersection point, and

♂-Z to zoom on the intersection point.

Repetition of the last five steps, using 10 for X-MAGNIF and Y-MAGNIF each time, produces increasingly accurate approximations of the intersection point. Each time you zoom, plot a segment of the graph of the current function, F or G. Next, press ♂-F to switch to the other function, plot a segment of its graph, and project the intersection point. Then zoom again.

When you are satisfied with the approximate X- and Y-coordinates displayed at the top of the screen, use Esc to exit the Project/Plot routine. You can tell how accurate X is by comparing the left side bound of the coordinate window with the right side bound. Similarly, the error in the Y-value is at most the difference between the top and bottom bounds.

If you continue zooming until the Project/Plot routine is exited automatically, you will get X = 1.5052415 and Y = 2.5052415 as the coordinates of the intersection point.

Using ♂-F to change functions while projecting is convenient in many applications. You can switch to any of the six functions (F, G, H, R, S, Q) to plot the graph and project values for comparison with another function.

Checking the Setup, ♁-S

To check on the current expressions for all parameters, commands and Functions, press ♁-S, select STATUS, and press Return. The first of four Setup/Status displays appears. Press the space bar to cycle through these displays. You can also press → to move ahead and ← to move back. Press Esc to exit.

Parameter Expressions

The first display shows all the current parameter expressions. If you have just started or used ♁-R to restart, most expressions are set at zero. Notice, however, that the expression for P is P = 3.141592654, a ten-digit expression for π. It has a label, PI.

It is important to remember that this display shows the current *expressions* for parameters, not necessarily their current *values*. For example, if the current expression for A is A = B + C, that is what appears on the parameter expressions display, regardless of the value of B + C. The current value of A (and all other parameters) is available on the Calculator display as a 9-digit decimal approximation. Of course, if the expression for A is a 9-digit decimal expression, then the expression and the current value appear to be identical.

Notice that the ten-digit expression for π given above produces the 9-digit current value P = 3.14159266 on the Calculator display. This displayed value is wrong in the last digit, but it has the advantage that SIN(P)=0. If you try it, you will see that SIN(3.14159265) ≠ 0.

If the definition of a parameter includes a label, this also appears following the expression. For example, if D is defined as

D = B * B – 4 * A * C; DISCRIMINANT

then this entire expression is displayed, including the label.

Also, notice that the variables X, Y, and T appear at the bottom of this first display. Since variables have no current expressions, their current values are displayed.

Command Expressions and Current Values

Press the space bar to see the next display. This display begins a listing of command *expressions* and their *current values*. The expressions appear at the top of the display, and the current values at the bottom. This display shows expressions and values used to control the coordinate window on the Graphics display. Notice that each expression is referred to by its menu name. For example, when you use ♁-W and select BOUNDS to change the window coordinates, you are prompted to enter expressions for Left Side, Right Side, Bottom, Top, X-Tick, and Y-Tick, in that order. Then the values of these expressions are used to define a new coordinate window.

These expressions and values are changed whenever ♁-W is used to change the window coordinates. The X-INT LEFT and X-INT RIGHT expressions and values are changed using ♁-I to change the X-interval choice for plotting graphs of functions.

And they are changed automatically when ⌘-W is used to change the coordinate window.

Press the space bar to continue the listing of command expressions and current values. The next display begins with T-INT LEFT and T-INT RIGHT. These choices are made with ⌘-I also, but in Parametric format where the T-interval is changed. (See *Function and Parametric Graphs, ⌘-F,* to learn more about graphing parametric equations). NUM POINTS refers to the number of points used to plot graphs when ⌘-G is used to plot. You can change the expression for NUM POINTS by using ⌘-N.

The next pair of expressions, X-MAGNIF and Y-MAGNIF, defines the X- and Y-magnification factors used in ⌘-Z, Zoom, a second-level command of ⌘-P, Project/Plot. The final three expressions Delta-T, Delta-X, and Delta-Y refer to the T, X, and Y increments used in several command routines. For example, Delta-T is used as the T-increment in projecting graphs of parametric equations and in displaying arrays of values for the corresponding functions X(T) and Y(T). Similarly, Delta-X and Delta-Y are used to change coordinate windows in the ⌘-W routine when you select FRAME. The commands used to change these three expressions are ⌘-T, ⌘-X, and ⌘-Y, respectively.

Function Definitions

Press the space bar again to see the last Setup/Status display. This display shows the definitions of eight functions. In Function format, F, G, H, R, S, and Q are used as functions of the variable X. The functions X(T) and Y(T) are not available for use in Function format. In Parametric format, all eight functions are available as functions of the variable T. But only X(T) and Y(T) are used for plotting graphs and displaying arrays of values.

Any labels appended to the definition of a function are also displayed on the Setup display. For example, if F is defined by

F(X) = M * X + B; LINEAR FUNCTION

then the label "Linear Function" is displayed.

Now, press Esc to return to the display where ⌘-S was initiated. You can step quickly through the Setup displays whenever you need to see a summary of the current Setup/Status. Just press ⌘-S and Return at the ? prompt.

You can save entire setups (all current expressions and/or values you see on the Setup displays) on a disk for loading again. (See *Loading and Saving Setups, ⌘-S*). Entire setups and/or graphics displays can be reloaded, or parts of the setup can be selected. This saves a lot of typing and allows for you to plan elaborate setups in advance and implement them quickly; or, you can save the details of important results and reproduce them later.

Scrolling Tables of Values, ⌘-A

The Array display provides 9-digit tables of values for any of the six functions (F, G, H, R, S, Q) whose definitions appear at the top of the display. These are the same functions used on the Graphics and Calculator displays. (Also see *Function and Parametric Graphs, ⌘-F* for arrays of values with parametric equations.)

The Array display is a numerical analogue of the Graphics display. On the Graphics display, ⌘-P, Project/Plot is used to plot the graph and project values of a function. The increment used is Delta-X. The currently projected point is emphasized by either lines or arrows. On the Array display, the same Delta-X is used to determine the X-values, and the current value of X is emphasized by a reverse video cursor. The current value of X is common to both the Graphics and Array displays. This makes the Array display convenient to use in conjunction with the Graphics display.
If you have not used the Array display before, review Examples 11 and 14 in Part I of this manual.

Solving Equations Numerically

Another use of the Array display if for solving equations numerically. The process is intuitive, and very little algebraic manipulation is needed. Further, transcendental equations can be solved quickly, even when algebraic methods fail.

Example 43: Suppose you want to solve the equation $e^x = x + 2$. An equivalent graphic problem is to find the intersection points of the graphs of the functions $f(x) = e^x$ and $g(x) = x + 2$. To begin, press

⌘-R to restart with the default setup,
F and define F(X) = EXP(X),
G and define G(X) = X + 2,
⌘-G to graph G, and
⌘-G to graph F.

Notice that there are two points of intersection. Use ⌘-P, Project/Plot to get a preliminary estimate for the right-hand intersection point. Just press ⌘-P and select Lines and F. Then use ← and → to approximate the coordinates of the intersection point. To improve this approximation, press ⌘-X to change Delta-X to .25E – 1, and adjust X to approximately 1.15. This is a reasonable preliminary estimate.

Now press Esc to end the Project/Plot routine. We are trying to solve the equation F(X) = G(X). The best way is to search for a zero of H(X) = F(X) – G(X). So, press

H and define H(X) = F(X) – G(X),
⌘-A to select H, and
← or → to adjust the array cursor to the smallest value of H(X).

The smallest value of H(X) (nearest 0) may be positive or negative; it occurs where the algebraic sign of H(X) changes between negative and positive.

Now, press ₲-X to change Delta-X to .25E – 2. When you press Return, the Array is regenerated with this smaller increment and the last cursor value for X is in the center. Again, use ← or → to locate the sign change in H(X). Then, change Delta-X to

.25E – 3. When the table is regenerated, find the sign change and change Delta-X to .25E – 4.

You can continue this process until Delta-X is reduced to .25E – 8. Notice that the sign change now occurs for approximately X = 1.14619322 and H(X) = 2E – 09. You can further reduce Delta-X to .25E – 9, but the displayed values of X might not improve. The increment eventually becomes too small to affect the value of X when added to it. (See Appendix 10, *Computer Arithmetic Errors, Negligible Addition.*)

You have found that X = 1.14619322 is a 9-digit approximation to one solution of $e^x = x + 2$. You can use this same method to determine that the other solution is approximately X = –1.84140566. It follows that the corresponding intersection points of the graphs are approximately (1.14619322, 3.14619322) and (–1.84140566, .158594340).

You may recognize the method of the last example as a generalization of the bisection method for finding zeros of functions. But it is more efficient than the bisection method for several reasons. First, using graphs with the Project/Plot command is a quick way to obtain preliminary estimates to pass to the Array routine. On the Array display, you observe eleven function values at a time and can quickly scroll to the values where there is a sign change. Finally, you can exercise judgement in choosing the function value closest to zero and (depending on this value) in choosing the size of the next increment, Delta-X.

As you gain experience, this graphic/numeric technique becomes even more efficient, because your judgement in selecting Delta-X improves. Often, you can obtain 8-digit approximations in four or five steps.

This graphic/numeric technique is also very general. It applies to a wide variety of functions and equations. Just enter the functions with BASIC syntax. If they are continuous, the method applies.

An alternative to using the Array display for finding the zeros of H(X) is to find them graphically using ₲-Z, Zoom to zoom in on the zero. Or, you can zoom on the intersection points directly. (See Example 42.) You may wish to repeat the last example with this method.

Displaying Current Parameter Values

Parameters can be defined on the Array display. Their current values are printed at the right side of the display, just as on the Graphics display. Whenever a function is defined using parameters in the definition, the current values of any parameters used are also displayed. If the last character for a displayed parameter value is a question mark (?), the displayed value has been truncated to five characters. However, the full 9-digit decimal value is used in all calculations. The truncation is just for display purposes. This is the same convention used on the right hand table of the Graphics display. You can see the full 9-digit value on the Calculator display.

Numerical Investigation of Functions

Numerical investigation of functions is often quite instructive. You can investigate function behavior near zeros or asymptotes numerically as well as graphically. Here is a simple example about the numerical meaning of slope:

Example 44: The purpose here is to examine the effect of M upon the values of a linear function, F(X) = M * X + B. First, press

Ó-R to restart. Then,
Ó-M to select Array mode,
F to define F(X) = M * X + B,
M to define M = 2,
B to define B = 1,
Ó-X to enter 1 for Delta-X, and
Ó-A and select F to generate an Array.

Notice that the current values M = 2 and B = 1, appear at the right side of the display. (See Figure 9.) Also, notice that the difference in consecutive F(X)-values is 2 times the difference in consecutive X-values reflecting the fact that M = 2. Scrolling the table through 30 or 40 values doesn't change this difference.

```
F(X)=M*X+B
G(X)=0
H(X)=0
R(X)=0
S(X)=0
Q(X)=0

                                      M=2
     X                  F(X)          B=1
-------------------------------
-5             -9
-4             -7
-3             -5
-2             -3
-1             -1
0              1
1              3
2              5
3              7
4              9
5              11

       PRESS ARROWS TO SCROLL
```

Figure 9.

Now, press Ó-X and change Delta-X to 2. A new table is generated, but still the F(X)-difference is twice the X-difference. Press Esc to return to the **?** prompt. Then press B and enter B = 5, and use Ó-A to generate another Array. Still, the difference in F(X)-values is twice the difference in X-values. Pressing X to enter X = 1000 shows that the difference is maintained for X-values near 1000.

Now, press Esc, and press M to change the value to M = 3; use Ó-A to generate a table. The F(X)-values differ by 3 times the X-difference, as expected.

This same property of linear functions can be explored graphically using ♂-P to project values on appropriate coordinate windows. Then, the geometric interpretation of slope can be introduced.

Press Esc to end this example.

After using ♂-A to generate an Array, only the ♂-X, X, ←, →, ↑, ↓, and Esc keys have any effect. Each time ♂-X or X is used, the table is regenerated, using the new value of Delta-X or X. The ↑ and ↓ or ← and → keys scroll the table. The Esc key returns you to the ? prompt.

At the ? prompt in the Array mode, any first-level control command can be used—just as in the Graphics and Calculator modes. If graphics commands like ♂-G, Graph Function or ♂-P, Project/Plot are used, the mode is changed automatically to Graphics mode.

Functions and parameters are defined in Array mode just as they are in Graphics or Calculator mode. Each time a function is defined, its new definition is displayed at the top of the Array display. As already mentioned, defining parameters or defining a function using parameters displays the current values of those parameters at the right side of the display.

It is important to note that at the ? prompt, the current values of X and the function—say, F(X)—can be assigned to parameters for use later. For example, press U and define U = X to save the current value of X in U. Then press V and define V = F(X) to save the current value of F(X) in V. Next, press W and define W = G(X) to save the current value of G(X) in W.

You can list the Array display to the printer whenever the ? prompt is present. See the section *Printing Graphics Calculator Displays* for the details.

Examples 43 and 44 use relatively small values for Delta-X. Large values are also useful.

Example 45: The objective here is to investigate the values of the function

$$g(x) = \frac{(4x^3 + 2x + 1)}{(2x^3 - 4x + 2)}$$

as X gets very large. Note that the BASIC form of this expression is as follows:

G(X) = (4 * X^3 + 2 * X + 1)/(2 * X^3 – 4 * X + 2)

Press

G and define G(X) as indicated,

♂-X and enter 10 for Delta-X,

X and enter X = 0,

♂-A and select G to generate an array, and

→ to scroll the table to larger values of X.

It appears that G(X) approaches the value 2 as X increases. At first, the difference between G(X) and 2 decreases rapidly, but it slows down considerably as X exceeds 500. To speed it up, press ♂-X and increase Delta-X to 100. Then press → to increase X to

about 5000. Again, the rate of convergence of G(X) to 2 has slowed. Now press ₲-X and increase Delta-X to 1000.

As X passes 32000, the value of G(X) becomes constantly 2. Of course, these are not exactly the correct values of G(X), which is never exactly equal to 2; but this is the best approximation to G(X) we can get on this computer. (See Appendix 10, *Computer Arithmetic Errors.*)

The above calculations show that Y = 2 is a horizontal asymptote for G(X), and that G(X) approaches 2 from above. They also give an indication of the rate at which the graph approaches the asymptote. If you change the numerator of G(X) to 4 * X^3 + 2 * X^2 + 1, then G(X) approaches 2 more slowly because of the quadratic term. You might also want to try some large negative values for X and observe that the graph approaches the asymptote Y = 2 from below.

Press Esc to end this example.

To investigate vertical asymptotes numerically, a small value is used for Delta-X. An initial estimate of the location of the asymptote is obtained graphically or numerically. Here is an example:

Example 46: The function

$$g(x) = \frac{(x + 1)}{(2x^3 - 4x + 2)^8}$$

has a vertical asymptote at X = 1 because the denominator is zero there. Note that the BASIC version of this expression is as follows:

G(X) = (X + 1)/(2 * X^3 – 4 * X + 2)^8

First, define G(X) as indicated. Then press

X to define X = 1,
₲-X and enter .25 for Delta-X, and
₲-A to generate an array for G.

Notice the tremendous range of values for G(X) in this array. In particular, G(X) is undefined for X = 1 and has large values near 1. To check values closer to 1, press ₲-X, and enter 1E –5 for Delta-X. (Notice the size of the function values in this table.) Then press ₲-X and enter .9E –5 for Delta-X. This time, some of the values are so large that *overflow* occurs. And if you make Delta-X even smaller, you get several values of G(X) UNDEFINED. This is the result of *underflow* in the denominator. That is, the denominator is rounded to 0. (For more on overflow and underflow, see Appendix 10, *Computer Arithmetic Errors.*) Press Esc to end this example.

The Array display complements the Graphics display by allowing comparison between graphic and numerical behavior. For more elaborate calculations, the Calculator display is used.

The Calculator Mode, -M

The Calculator mode is designed to complement the Graphics and Array modes. Since functions, parameters, and variables are defined globally, those used in one of the three modes are also available in the other two modes. On the other hand, the Calculator is useful in its own right for doing calculations involving a large number of values, or for calculating with several formulas at once. Iterative calculations are also convenient. For many applications, it is faster to use the three modes of the *Graphics Calculator* interactively than to write and debug your own computer programs.

Before proceeding with this section, you might want to review the chapter titled *The Calculator Mode* in Part I of this manual. For more details, see Appendix 2. You should also review the chapters *Defining Functions* and *Defining Parameters* in Part II before proceeding.

To continue this discussion, press -R to restart with the default setup and -M to change to the Calculator mode. If you have reviewed *Defining Parameters*, you are familiar with this display and you know how the definition of parameters affects their current values and current expressions. Since you have just used -R to restart, all current values and expressions (except P) are zero (see Figure 10).

```
F(X)=0
G(X)=0
H(X)=0
R(X)=0
S(X)=0
Q(X)=0


A=0           L=0
B=0           M=0
C=0           N=0
D=0           O=0
E=0           P=3.14159266
I=0           U=0
J=0           V=0
K=0           W=0

                        ?
```

Figure 10.

The BASIC Design

The principle on which the Calculator mode works is quite simple. It uses the assignment statement of BASIC to assign *current values*. You can see the current values of all 15 parameters at once. In effect, you are looking at the values in 15 storage registers. These values are updated using BASIC arithmetic expressions. Each register has its own *current expression*. That expression uses values from any of the registers to calculate and assign new current values. To see a current expression, just press the corresponding letter. The current expression appears in the display at the bottom of the screen. If you press Return, the current value is updated. If you do not want to

update the expression or current value, press Esc . To see all expressions at once, press ✇-S and select "Status" to observe the first of the Setup/Status displays. Then, press Esc to return to the Calculator display.

The design of the Calculator mode makes it more useful than an ordinary calculator. The input routine allows you to change expressions with a minimum of typing. Iteration is easy, because expressions can be reused without retyping them. Current values and expressions can be used in any order desired. Since you can see all current values at once, you can exercise judgement at intermediate stages of a calculation, or you can experiment and compare results. Syntax errors and undefined values are trapped immediately, so that you can recover before proceeding. You can get a hard copy of the Calculator display and corresponding Setup/Status displays on your printer. Also, you can save a complete setup—including all functions, parameter values and expressions, command values and expressions, and graphics display—on a disk for later use. (See *Loading and Saving Setups, ✇-S.*) Here are some examples.

Calculator Setups

The following two examples illustrate how to set up the Calculator for specific types of problems.

Example 47: Suppose you need to find the length, midpoint, and slope of several line segments using their endpoints. If the endpoints are (I,J) and (K,L) and the first pair of coordinates is (–3, 2) and (4, –3), we set up the Calculator as follows:

```
I = –3  ; 1ST COORDINATE
J = 2   ; 2ND COORDINATE
K = 4   ; 1ST COORDINATE
L = –3  ; 2ND COORDINATE
D = SQR((I – K)^2 + (J – L)^2) ; LENGTH
M = (J – L)/(I – K) ; SLOPE
U = (I + K)/2 ; MID POINT 1ST COORD.
V = (J + L)/2 ; MID POINT 2ND COORD.
```

Remember, the labels after the semicolon are optional. Notice that space is left before the label in I, J, K, and L to change expressions. As you enter this setup, the current value of each parameter is displayed at the appropriate place on the parameter tables. To see all expressions at once, press ✇-S.

Each time you change the current value of I, J, K, or L, you can recalculate D, M, U, or V without retyping the expression for the formula. For example, pressing D, Return updates the current value of D. Try several combinations of your own for I, J, K, and L. Recalculate D, M, U, and V for each change in I, J, K, and L.

To learn how to save setups (like the one in the last example) for reloading without retyping, see *Loading and Saving Setups, ✇-S* .

Now press ✇-R to restart and ✇-M to enter the Calculator mode.

Example 48: The compound and continuous interest formulas provide interesting comparisons. Enter the following setup:

C = 100 ;INITIAL INVESTMENT
I = .10 ;NOMINAL INTEREST RATE
N = 12 ;INTEREST PERIODS PER YEAR
T = 10 ;TIME IN YEARS
A = C * (1 + I/N)^(N * T) ;COMPOUND INT.
B = C * EXP(I * T) ;CONTINUOUS INTEREST
D = B – A ;DIFFERENCE

The value of D tells us that the advantage of continuous compounding over monthly compounding is only $1.12 in 10 years for $100 invested at 10% nominal rate.

To try other combinations, just change the current values of C, I, N, or T. Then update the current values of A, B, and D (in this order). Notice that the remark for T is not reproduced each time, and the current value for T is not displayed in the parameter table. This is because T is a variable, not a parameter. Variables do not have current expressions. To see the current value, you must press T.

Zeros of Functions by Iteration

In Examples 47 and 48, parameters are updated at your discretion, depending on the calculations and comparisons in which you are interested. The next two examples are different. They are examples of iteration methods for finding zeros of functions. "Iteration" means that a process or sequence of operations is repeated until a desired result is achieved or some other conclusion has been reached. In these examples the desired result is an approximation to a zero of a function.

Example 49: Many precalculus mathematics textbooks discuss the bisection method for finding zeros of functions, F(X). The function must have the *intermediate value* property. This means that if F(I) and F(J) have opposite algebraic signs, there is a zero of F between I and J. Polynomial functions have the intermediate value property.

Here we intend to approximate, using the bisection method, a zero of

$$f(x) = 2x^3 - 3x^2 + x + 1$$

which in BASIC is represented as

F(X) = 2 * X^3 – 3 * X^2 + X + 1

(We could find it more quickly using the graphic/numeric methods shown earlier, but this example is for demonstration purposes).

First, press ♂-R to restart and ♂-M to return to the Calculator display.

Now, press F and define F(X) as indicated above. Then check A = F(-1) and B = F(0); notice that F(X) is *negative at the left endpoint and positive at the right endpoint* of the interval [-1,0]. This information is used below.

To begin the bisection iteration, define

I = -1 ; LEFT ENDPOINT

J = 0 ; RIGHT ENDPOINT

M = (I + J)/2 ; MIDPOINT

N = F(M) ; MIDPOINT FUNCTION VALUE

Since we started with F(X) negative at the left endpoint of [I,J] and positive at the right, we want to maintain this situation. So if N is negative, as it is now, define I = M to update the left endpoint. Then press M, Return and N, Return to update M and N. Now N is positive, so define J = M (to keep F(X) positive at the right endpoint) before updating M and N again.

Now enter I, M, N (when N < 0) or J, M, N (when N > 0) repeatedly, until the values of I and J agree to four digits. That is,

I = M ;LEFT END POINT IF N < 0

J = M ;RIGHT END POINT IF N > 0

M = (I + J)/2 ;MID POINT

N = F(M) ;MID POINT FUNCTION VALUE

After about ten repetitions, the values of I and J agree to four digits, and the zero of F(X) is between I and J. This means that –.39816 is a 5-significant-digit approximation to the zero of F(X). (See Figure 11.)

```
F(X)=2*X^3-3*X^2+X+1
G(X)=0
H(X)=0
R(X)=0
S(X)=0
Q(X)=0


A=0                 L=0
B=0                 M=-.398159027
C=0                 N=8.35279934E-06
D=0                 O=0
E=0                 P=3.14159266
I=-.398162842       U=0
J=-.398155212       V=0
K=0                 W=0


   N=F(M)                         ?
```

Figure 11.

The previous example shows how the bisection method works. For instructional purposes, it is often enlightening to follow a procedure step-by-step in this way. But as a practical method for finding zeros accurately, the bisection method is too slow to use by hand. The next example demonstrates the secant method. This method usually requires fewer iterations, and you don't have to make a decision about end points.

Example 50: To begin, press ⌘-R to restart. Then define $f(x) = x^3 - 3x - .5$ [or, in BASIC, F(X) = X^3 – 3 * X – .5].

The secant method requires two preliminary estimates of the zero. The better these estimates are, the fewer iterations will be required. Good estimates are obtained graphically using ⌘-P, Project/Plot to project the intercept and store the X-coordinate. Here is how it is done:

Press ⌘-P and select Lines and F.
Then, use ← and → to plot the graph.
Move X to approximate the zero near 1.75 .
Press ⌘-X and change Delta-X to .25E –1 .
Adjust Y as near to 0 as possible.
Press B and assign B = X.
Press → to move X slightly.
Press A and assign A = X.
Press Esc to exit Project/Plot.
Press ⌘-M and select Calculator.

Now the values of A and B are approximately 1.8 and are good preliminary estimates for the zero of F near 1.75. Next, use the secant formula to get an improved estimate. That is, define

$$c = b - \frac{(a-b)}{f(a) - f(b)} \cdot f(b)$$

or in BASIC,

C = B – (A – B)/(F(A) – F(B)) * F(B)

Then, repeat the following three steps several times. Three or four iterations are sufficient to obtain C = 1.81003793 as a good approximation to the zero.

A = B
B = C
C = B – (A – B)/(F(A) – F(B)) * F(B)

Each time C is updated, check to see if B and C have the same value. If so, you may stop. Continuing may eventually result in F(A) = F(B). Then the value of C is undefined due to division by zero.

The secant method converges quickly to simple zeros. It is not very efficient at multiple zeros, however. For example, successive approximations to the double zeros of

$$f(x) = x^4 - 4x^3 + 8x + 4$$

or, in BASIC:

F(X) = X^4 – 4 * X^3 + 8 * X + 4

improve slowly with the secant method (and most other methods too). The graphic/numeric methods described in earlier sections are better. Or, if you know some calculus, you may find the zeros of the derivative.

It is instructive to perform iterations by hand and to observe the intermediate results. But after a method is learned, it is better to have it operate automatically. *Graphics Calculator*

has features to accomplish this also (See *Programming Graphics Calculator,* ♂-*O* and Examples 66–70.)

If you know some calculus, you might want to repeat the previous example using Newton's method. Only one preliminary estimate, A, is required. Then, define B = A – F(A)/G(A), where G(X) = 3 * X^2 – 3 is the derivative of F(X). Now, repeat the two steps:

A = B
B = A – F(B)/G(A)

until A and B have the same value.

Loading and Saving Setups, ⌘-S

When you press ⌘-S, the menu

```
SETUP: [STATUS] LOAD SAVE CAT DEL PRNT
```

is presented. For a discussion of the first selection, STATUS, see the section entitled *Checking the Setup, ⌘-S.* The following selections are used to load, save, catalog, or delete setups. The last selection is for printing graphics displays.

When you select SAVE, all current expressions listed on the ⌘-S, Setup/Status displays are saved on your auxiliary disk. Also, all corresponding current values are saved. This means that all parameter expressions and values, all variable values, all command expressions and values, and all function definitions are saved. Also, you have the option of saving the Graphics display. And you may supply a title with each setup, which will be printed at the bottom of the screen when the setup is reloaded. These saved Graphics displays may later be reloaded and printed. With printer systems supported by *Graphics Calculator*, the Graphics display may be printed without first saving it on an auxiliary disk.

When you select LOAD, you have the option of loading all or just parts of a setup. For example, you can load the functions, parameters, or commands separately; or you can load just the Graphics display and commands together. This versatility in setup allows you to load a collection of functions without disturbing the parameter setup or the graphics setup. Or, if necessary, you can load an entire setup and a graphics display. If a setup was saved with a title or message, the title or message will be printed at the bottom of the current display whenever all of part of the setup is loaded.

The ⌘-S, Setup command is especially useful for preparing presentations in advance. Complicated setups can be prepared, saved, and then loaded at the appropriate time. Setups can be loaded or reloaded in any order so you can respond to questions or unexpected situations by repeating setups. With very little typing, you can load a well-designed, error-free setup, use it immediately, then change to a different setup.

You can also use ⌘-S, Setup to keep a record of your work as you proceed. For example, when a final solution to a problem is obtained, the entire setup resulting in that solution can be saved and checked later.

You can save from 10 to 50 setups on an auxiliary disk depending on whether or not you save the Graphics displays. Graphics displays use almost five times as much disk space as the rest of the setup, so you may want to save only those for which you want hard copy. You can reproduce many Graphics displays on the screen quickly simply by loading a setup and plotting a few graphs, so it is often more economical in terms of disk space not to save Graphics for later display on the screen.

To save a setup, you will first need an auxiliary disk on which to save it.

Making Auxiliary Disks

If your main *Graphics Calculator* disk is write-protected (as it should be!), you cannot save setups on it. Instead, you will need an auxiliary disk. You can make an auxiliary

disk simply by copying the auxiliary disk you received with *Graphics Calculator*. Use your favorite disk-copy program.

Saving a Setup

You can press ♂-S to save the current setup whenever the ? prompt is present. Before beginning the following examples, insert your main *Graphics Calculator* disk into drive 1, and load *Graphics Calculator* in the usual way. When booting is complete and the ? prompt appears, remove the main disk and insert an auxiliary disk into drive 1. (An alternative, if you have two disk drives, is to leave the main disk in drive 1, insert the auxiliary disk into drive 2, and press ♂-D to change the "Disk Drive in Use" selection to 2. Then the computer reads and writes to drive 2 until you change the setting or restart.)

Example 51: We start with a simple but useful example—saving the default setup. You will see why it is useful when Load is discussed. You should have an auxiliary disk in the drive and be at the Graphics display with the default setup. If not, insert an auxiliary disk and press ♂-R to restart. (If your auxiliary disk is in drive 2, press ♂-D to reset the drive to 2.) To save the default setup:

Press ♂-S, select SAVE and press Return.

Type RESTART for the filename, and press Return.

Select YES for SAVING GRAPHICS? and press Return.

Select YES for SUPPLYING A TITLE? and press Return.

Type RESTART SELECTION SET TO DEFAULT for a title, then press Return.

When you press the last Return, the title is centered, and the default setup is saved on the auxiliary disk under the filename restart. The Graphics display is saved because you selected Yes. The title you supplied is printed at the bottom of the display each time you load the file RESTART. In the next example, you will see how to use RESTART.

Filenames and Title Characters

There are restrictions on the characters you can use for filenames and titles. Filenames must begin with a letter and may contain only letters of the alphabet, digits, periods or hyphens. These filenames may contain no more than eight characters.

Titles may contain any character the input routine accepts (see Appendix 1). The titles are limited to 34 characters and are centered automatically. If you do not want the title centered, use spaces before or after it to affect its location.

Setup Catalog

If you press ♂-S and select CAT, a catalog of Setup filenames is displayed, with a Yes or No in the right-hand column, depending on whether or not graphics are saved with that file. Saved graphics can be printed; the method depends upon your printer. The ♂-S menu appears at the bottom so that you can make a selection while looking at the catalog. If you select DEL, the filename entered is deleted from the disk and catalog, and you return to the catalog. As usual, to exit a menu without making a selection, press Esc. It is a good idea to make filename entries while looking at the catalog, so that you do not inadvertently reuse a filename or delete the wrong file. To delete just the Graphics display, load the file and resave it without the Graphics.

Loading a Setup, Customized Restart

The following example illustrates how to load a setup. At the same time, it illustrates the convenience of using your own restart setups.

Example 52: To see why saving the RESTART file in Example 51 is useful, press ⌘-S, select Load, and enter RESTART for the filename. The disk drive runs briefly, as the catalog is checked, to see if your filename is on the current auxiliary disk. If it is, the following menu appears:

LOAD: [ALL] FUNC PAR CMND GRPH

Selecting ALL loads functions, parameters, commands and the Graphics display. Selecting FUNC, PAR, or CMND loads only the functions, parameters, or commands, respectively. Selecting GRPH loads both the Graphics display and commands. (Considerable confusion can result if the command setup does not agree with the graphics at which you are looking!) If a file has no Graphics, the GRPH selection is not offered.

Make the selection you want on the Load menu, then press Return. The disk drive runs again as the setup is loaded. Then your title, RESTART SELECTION SET TO DEFAULT, is printed at the bottom of whatever display (Graphics, Array, Calculator, or Help) you were at when ⌘-S was pressed. To see what gets loaded with 4each of the FUNC, PAR, and CMND selections, press ⌘-S and check the corresponding STATUS displays.

The ability to change just parts of a current setup to default values makes saving the restart file useful. Often, you want to keep part of a setup—perhaps just the parameters and functions—but you want to start over with the default coordinate system. In this case, load only commands from restart.

You probably won't select ALL very often with restart, because it is faster to use ⌘-R, restart to get the entire default setup. On the other hand, if you have saved your own personal default setup with some different default settings, you may use ALL frequently. You may also choose not to save the Graphics display. It uses considerable disk space and usually is not needed.

SETUP affects only those commands whose expressions are listed on the Setup/Status display. The settings of those like ⌘-C, Color of Graph or ⌘-D, Disk Drive in Use are not changed by a load. There is one exception: the setting of ⌘-F on Function or Parametric format is saved and reloaded.

To set a graph color or disk drive automatically, you must save an O-Sequence with appropriate commands and "run" it after loading the setup.

A Quadratic Function Setup

Here is a more elaborate example using setup:

Example 53: As seen in Example 16, the following function and parameter setup is useful for analyzing quadratic functions and their graphs. It takes a little time to enter, especially if you include the labels for each expression, but you only have to do it once!

If you have just finished Example 16, you have already entered the setup. If not, press ⌘-M to get to the Calculator display, and enter the following functions and parameters as indicated (or you may load the QUADRATIC setup from the main disk.):

F(X) = A * X^2 + B * X + C; QUADRATIC
A = 1 ; LEADING COEFFICIENT
B = –4 ; LINEAR COEFFICIENT
C = –5 ; CONSTANT TERM
D = B * B – 4 * A * C; DISCRIMINANT
I = (–B + SQR(B^2 – 4 * A * C))/(2 * A); ZERO
J = (–B – SQR(B^2 – 4 * A * C))/(2 * A); ZERO
U = –B/(2 * A) ; X-VERTEX COORDINATE
V = F(U) ; Y-VERTEX COORDINATE

To check all the above parameter expressions at once, before you save them, press ⌘-S to look at the first Status display. Press Esc to return to the Calculator display.

To save this setup, proceed as follows:

Be sure you have an auxiliary disk in drive 1,
press ⌘-S and select SAVE,
enter QUAD for name of file,
select No for ARE YOU SAVING GRAPHICS?
select Yes for ARE YOU SUPPLYING A TITLE?, and
for a title, enter TO SEE QUAD SETUP, ENTER APPL-S.

When you press the final Return to enter the title (which in this case is a message to the user), the setup is stored on the auxiliary disk. If you check the catalog (use ⌘-S, CAT), it is listed there with filename QUAD. The setup is still in computer memory too, so you can use it now without reloading it. For example press ⌘-G to plot the graph of F.

Switch to any display, and you will find that it uses the new setup. Loading a setup does not switch you automatically to the display from which the setup was saved. However, if a setup is saved from Parametric format, you are automatically in Parametric format when it is reloaded, and at the display from which loading started. (See the section *Function and Parametric Graphs, ⌘-F.*)

Example 54: To experiment with the ⌘-S, Load option, press ⌘-R to restart with the default setup. Then,

press ⌘-S and select Load,
enter QUAD for name of file,
select one of ALL, FUNC, PAR, or CMND from the Load menu (and press Return),
press ⌘-S again, and
select Status to check the four Setup/Status displays.

Repeat the above sequence of commands several times, each time pressing ♂-R first to restart. Then press ♂-M to switch to a new display, and try loading Quad from there. You may want to plot the graph of F for a few combinations of A,B, and C on the Graphics display.

A Trigonometric Setup

The setup in the following example is used to investigate the trigonometric functions and their graphs.

Example 55: Be sure you have the main *Graphics Calculator* disk in drive 1. Then,

press ♂-M to switch to Calculator mode,
press ♂-S and select CAT for the catalog,
select Load from the Setup menu,
enter TRIG.FUNC for the filename, and
select ALL from the Load menu.

The following functions are loaded and appear on the Calculator display:

F(X) = A * SIN(B * X + C) ; SINE FUNCTION
G(X) = A * COS(B * X + C) ; COSINE FUNCTION
H(X) = A * TAN(B * X + C) ; TANGENT FUNCTION
R(X) = 1/TAN(X) ; COTANGENT FUNCTION
S(X) = 1/COS(X) ; SECANT FUNCTION
Q(X) = 1/SIN(X) ; COSECANT FUNCTION

Notice that the cotangent, secant, and cosecant functions must be defined using the tangent, cosine, and sine functions because they are not part of the BASIC built-in functions set. (Appendix 4 contains definitions for other functions not directly provided by BASIC.)

Now, press

A to define A = 2,
B to define B = 1, and
♂-G to graph F.

Notice that the coordinate system loaded with the setup is appropriate for graphing some trigonometric functions. It is defined in terms of π in the X-direction. To check the setup further, press ♂-S to see the Setup/Status displays. The second display shows the coordinate system defining expressions and values. The third display shows that the number of points used to plot has been increased to 86, and that the X-increment, Delta-X, is P/16. Remember that P = π. Press Esc to exit the Setup/Status displays.

If you want to save a different trigonometric setup, you can alter the current setup and save the altered version with a different filename on an auxiliary disk.

You may want to plot the graphs of some of these functions using various values for A, B, and C before ending this example; or you can do it at a later time simply by loading the setup.To avoid having asymptotes plotted for the functions H, R, S, and Q, plot their graphs unconnected (♂-U).

Printing *Graphics Calculator* Displays

Printing Text Screens

You may print any text display on *Graphics Calculator* if your printer card is in slot 1. Your printer must be on line and have paper in it. At the appropriate time, press ↺-L to initiate the text-printing routine.

The appropriate times to press ↺-L are as follows:

- On the Array, Calculator, and Help screens, press ↺-L when the ? prompt is present.
- On the ↺-S, Setup/Status display, press ↺-L when the "Press ← or → for more (Esc exits)" message appears.
- On Catalog displays, press ↺-L when the setup or O-Sequence menu returns at the bottom of the display.
- If you press ↺-L at the Graphics display, you are presented with a blank text screen that you probably do not want to print. You can press Esc to return to the ? prompt.

When you are printing, a changing character flashes at the center of the last text line on your screen. When printing is completed, this character disappears, and you may continue with normal operation.

Titles for Text Displays

If you press ↺-L, select Yes, and press Return, you then have a choice of supplying a title or not. If you select Yes again, the bottom line is erased, and the input cursor appears so that you can type your title. All of the features of the input routine work here. (See "Entering and Changing Expressions" in the *Calculator Mode* chapter of Part I, and/or see Appendix 2: *Entering Titles, Filenames, and Algebraic Expressions*.) When your title is ready, press Return. The title is centered automatically, and the text display is printed. If you don't want the title centered, you can alter its location by using blank characters (pressing the space bar) as you type. To move the title to the right, add blanks before it; to move it to the left, use blanks after it.

If you choose not to title your printout, just select No at the ARE YOU SUPPLYING A TITLE? menu. When you press Return, the text display is sent to your printer.

Printing Graphics Displays

If you have a graphics printer, you may be able to print Graphics displays while the *Graphics Calculator* program is running. Refer to Appendix 9 for specific information on printing graphics displays from *Graphics Calculator*.

Function and Parametric Graphs, ⌘-F

The previous sections of this manual are presented mainly in the Function format; most of the graphs plotted are graphs of functions. In this section, we introduce Parametric graphs. (A special case of Parametric graphs—Polar graphs—is also introduced.) We assume, in this section, that you are familiar with the Function format. Most control characters and corresponding commands used in the Function format produce similar results in the Parametric format.

Graphics Calculator should be in the Function format now. If it is not, press ⌘-F, select FUNCTION, and press Return. *Graphics Calculator* will restart in Function format.

Press ⌘-M to change to the Calculator display. *Graphics Calculator* is in Function format on each of the Graphics, Array, and Calculator displays. Notice that the six functions in the function table have the independent variable X. When you change to Parametric format, the independent variable is T.

Restarting in Parametric Format

Whenever you switch to Parametric format, *Graphics Calculator* will restart at the Graphics display with the default Parametric format setup. To do this, press ⌘-F, select Parametric, and press Return. The initial Graphics display appears just as it does in Function format.

Press ⌘-M to switch to the Calculator display. *Graphics Calculator* is in Parametric format on the Graphics, Array, and Calculator displays. All six functions in the function table have the independent variable T. Also, the functions S(T) and Q(T) are replaced by X(T) and Y(T) in this display. If you press ⌘-S to check the function table on the Setup/Status displays, you will find that S(T) and Q(T) are still available as functions of T. So, there are eight functions available in Parametric format. The functions X(T) and Y(T) are used for graphs and tables of values. The functions F, G, H, R, S, and Q are used mainly to help define X(T) and Y(T).

The Graphics Display in Parametric Format

In Parametric format, the Graphics display operates much the same as it does in Function format. However, there are some differences, as shown in the following example:

Example 56: First, define the functions X(T) and Y(T).

Press:

X to define X(T) = T • (T – A) • (T + A),

Y to define Y(T) = T, and

A to define A = 2.

Notice that pressing X or Y at the **?** prompt in Parametric format does not allow you to assign current values to the *variables* X or Y, because X and Y are now *functions.* Nor can you use the variables X and Y in expressions that define functions or parameters. You

can, however, use the functions X(T) and Y(T). You can assign current expressions and values to the parameters A, B, C, . . . , just as in Function format.

Press ♂-G and Return to graph X(T), Y(T). The Graphics display appears immediately, and the graph is plotted. Press ♂-P, select ARROWS, and press Return. The Project/Plot routine displays the X- and Y-coordinates of the projected point at the top of the display. The corresponding T-value is displayed at the bottom of the Graphics display on the right. A segment of the T-axis on the bottom left represents the current T-interval. The default interval is [–5, 5]. A projection arrow points to the current value of T in this interval. (See Figure 12.)

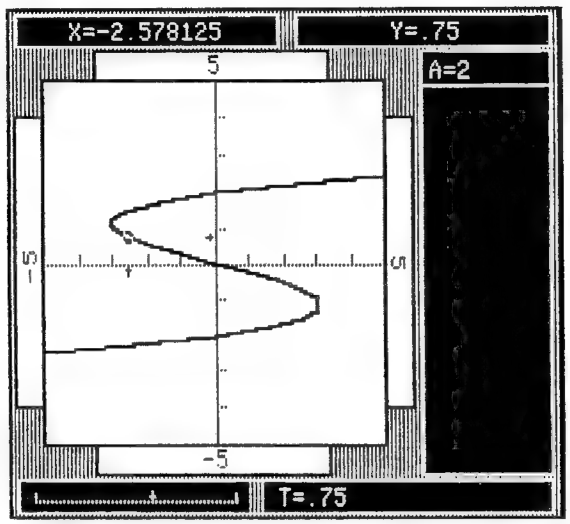

Figure 12.

As you press ← or →, the value of T changes by the amount Delta-T, the projection arrows all move to new positions, and the new X-, Y-, and T-values are displayed. If you hold down ← or →, the projections move quickly. Values are printed only when keys are released. The default values of T and Delta-T are 0 and.25, respectively.

To assign a specific value to T, press T and enter the value. Similarly, to change Delta-T, press ♂-T and enter the new value. For example, press T and enter T = 1; then choose Delta-T and enter .1. With a smaller Delta-T, the projections move more slowly.

To trace the X- and Y-values on the axes, press ♂-M for Mapping Trace, and use the ← and→ keys, just as in Function format. There is no trace on the T-axis. To deactivate the Mapping Trace, press ♂-M again.

To assign a projected value of X, Y or T to a parameter, simply press the corresponding letter. For example, press B. In the right-hand table, use ← or → to select X, Y, or T, and press Return to complete the assignment. [Do not use A here, because X(T) has an A in its definition. Changing the value of A changes your function!] The current value of X, Y, or T is assigned as the current *value* of B. The current expression for B is not changed.

To exit Project/Plot, press Esc, and the **?** prompt returns, as usual.

You can change the coordinate system in Parametric format just as you can in Function format. Press ✱-W and select "Bounds" or "Frame." The Bounds and Frame options work just as in Function format. You can also "Zoom" in Parametric format. While in the ✱-P, Project/Plot routine, the ✱-Z, Zoom option is similar to Function format. However, T and Delta-T (press T and ✱-T, respectively) are used in place of X and Delta-X to control the projection of the center of magnification. Delta-T is not changed automatically, since the best value depends on both X(T) and Y(T) and is therefore unpredictable.

The default T-interval is [–5,5]. This is changed using ✱-I, Interval Choice, just as in Function format. The only difference is that the T-interval, rather than the X-interval, is changed. The T-interval is not affected by using ✱-W to change Window Coordinates.

As usual, the number of points plotted is changed by using ✱-N, Number of Points. ✱-T, T-Increment is used at the ? prompt (or in the ✱-P and ✱-A routines) to change the increment, Delta-T. The same Delta-T is used by ✱-P, Project/Plot and on the Array display.

To see the current Setup/Status, press ✱-S. The current values of X, Y, and T appear at the bottom of the first display, the T-interval and T-increment are on the third display, and all eight functions are on the last display.

Circles and Ellipses

Circles and ellipses are easy to graph in Parametric format.

Example 57: It is convenient to plot the graphs of ellipses with the equation

$$\frac{(x-u)^2}{a^2} + \frac{(y-v)^2}{b^2} = 1$$

in parametric form. The parametric equations are

$$x(t) = u + a \cos(t)$$

and

$$y(t) = v + b \sin(t)$$

The point (U,V) is the center of the ellipse, 2a is the length of one axis, and 2b is the length of the other axis. (We assume that a > 0 and b > 0.)

To plot these ellipses, enter the following setup in Parametric format:

```
X(T) = U + A * COS(T); X-COORD. OF ELLIPSE
Y(T) = V + B * SIN(T); Y-COORD. OF ELLIPSE
A = 2 ; LENGTH OF ONE AXIS IS 2 * A
B = 4 ; LENGTH OF ONE AXIS IS 2 * B
U = 0 ; 1ST COORD. OF CENTER
V = 0 ; 2ND COORD. OF CENTER
```

Then press:

✱-I to change the T-interval to [0,2π],

✱-T to change Delta-T to 2π/40, and

✱-G to plot the ellipse.

Before plotting more ellipses, you might want to use ⌘-S, Setup to save this setup on an auxiliary disk for later use. Then try some different values for A, B, U, and V. Each time one of these parameters is changed, its new value is printed in the right-hand display. Notice how changing A or B affects the shape of the ellipse, while changing U or V affects its location. (See Figure 13.)

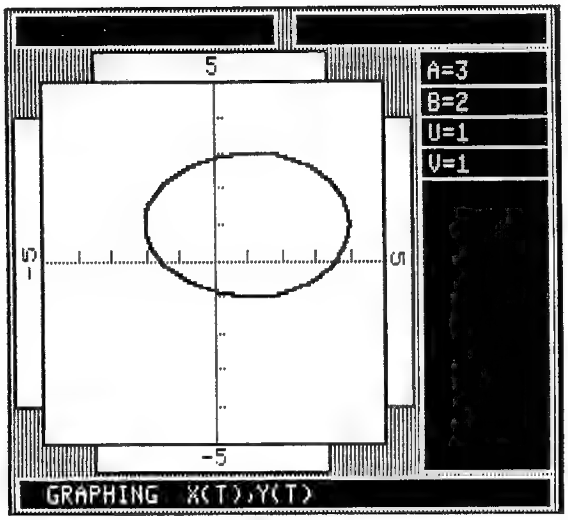

Figure 13.

Also, try ⌘-P, Project/Plot to project some coordinates of points on the ellipses. You may want to save the projected coordinates of the vertices and display them in the right-hand table. But be sure to assign the values to parameters other than A, B, U, or V, because changing their values changes the graph!

For another interesting experiment, use ⌘-I, Interval Choice to change the T-interval to [0,20π]. Then press ⌘-P to project values, but hold the arrow key ← or →, down. Notice the periodic motion of the projection arrows and how it is affected by changing A and B.

A few changes in the setup of the last example produces a "helix." The idea is to move the center continuously as ellipses are plotted.

Example 58: Modify the setup of the last example as follows:

X(T) = T/16 + A * COS(T),
Y(T) = T/16 + B * SIN(T),
A = 2, and
B = 2.

Then press:

⌘-I to change the interval to [–8π,8π],
⌘-N to change the number of points to 201,
⌘-T to change Delta-T to 16 * π/200, and
⌘-G to plot the graph.

As usual, it is a good idea to have Delta-T match the increment produced by ♂-N, Number of Points. The expression $16\pi/200$ is the length of the T-interval, 16π, divided by 200, which is one less than the number of points. (See Figure 14.) Changing A or B changes the shape of the "helix."

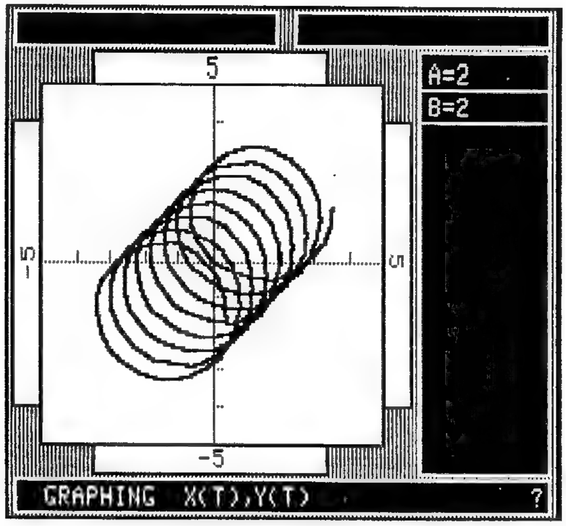

Figure 14.

Inverse Relations and Graphs

Another way to use Parametric format is to investigate inverse relations. For example, suppose you have defined the function F(T). If you then define X(T) = T and Y(T) = F(T), the parametric graph is the same as the graph of the equation Y = F(X). On the other hand, if you interchange the X and Y coordinates by defining X(T) = F(T) and Y(T) = T, the parametric graph is the graph of the inverse of F(X), $Y = F^{-1}(X)$ — or X = F(Y), which is equivalent. Try this with your favorite function F. Note that the second graph is the reflection of the first in the line Y = X.

Lissajous Figures

There are many interesting families of parametric curves. The lissajous figures are one popular family. The general definition of these curves is

X(T) = A * SIN(B * T + C)
Y(T) = U * SIN(V * T + W)

The use of about 100 points on the T-interval $[0,2\pi]$ usually produces a satisfactory graph. However, if B or V is greater than 10, use more points. The parameters, A and U, alter the size of the figure. Try A = 3, U = 3, C = 0, and W = 0 to start.

The most interesting figures occur when B and V are integers with no common factors. Try B = 5 and V = 7, for example. (See Figure 15.) The shape of the figure is sometimes affected by the number of points chosen. For example, try B = 16 and V = 17 with 65, 66, 67, 68, 69, 70, 71, 100 or 200 points to see different shapes.

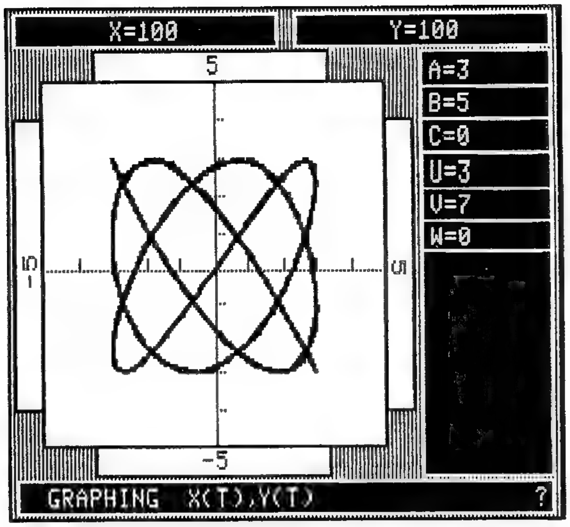

Figure 15.

Tables of Parametric Equation Values

The Array display in Parametric format shows eleven values of X(T) and Y(T), with the current values in reverse video. The table scrolls using ←, →, ↑ or ↓, just as in Function format. The current value of T appears at the bottom. Press T and ⌘-T to change the current value of T or Delta-T, respectively. Here is a simple example:

Example 59: Suppose you want to compare the effects of compound and continuous interest over several years. You can define

X(T) = C * (1 + I/N)^(N * T); COMPOUND

and

Y(T) = C * EXP(I * T); CONTINUOUS

Then scroll tables of values through the desired values of T to compare X(T) and Y(T). Here, T is time in years, C is the initial investment, I is the nominal interest rate, and N is the number of compounding periods per year.

To begin, press:

⌘-R to restart in Parametric format,
⌘-M to change to the Array display,
C to define C = 100,
I to define I = .08, and
N to define N = 4.

Notice that the current values of C, I, and N are printed at the right side of the display as they are defined. Then:

Define X(T) and Y(T) as indicated above.
Press ⌘-T to enter 1 for Delta-T.
Press ⌘-A to generate an Array.

The current value, T = 0, appears at the bottom of the display, and the corresponding current values of X(T) and Y(T) are highlighted in reverse video at the center of the table. The values above this are for negative T; they are discounted values. Values for positive T are accumulated values. See Figure 16.

```
F(T)=0
G(T)=0
H(T)=0
R(T)=0

X(T)=C*(1+1/N)^(N*T);COMPOUND
Y(T)=C*EXP(I*T);CONTINUOUS

   X(T)              Y(T)
--------------------------------
1.15292151       67.0320046
2.81474977       72.6149037
6.87194768       78.6627861
16.777216        85.2143789
40.96            92.3116347
100              100
244.140625       108.328707
596.046448       117.351087
1455.19152       127.124915
3552.71368       137.712776
8673.61738       149.18247

 T=0
```

Figure 16.

You can press ↑ or ↓ to scroll the table in either direction. To see values for a particular value of T, just press T and enter it. To change the increment, Delta-T, press -T and enter the new increment. Notice that for T = 10, the difference in X(T) and Y(T) is only about $1.75.

Press Esc to return to the **?** prompt.

Limits of Functions

You can use the Array display to investigate limits of functions. To observe values of F(X) as X tends to A, simply define

X(T) = A + 1/T

and

Y(T) = F(X(T))

Then scroll the Array for large values of T (so that 1/T is small). The values of X(T) tend to A, and Y(T) = F(X(T)) shows the behavior of F(X) for X near A.

Example 60: Suppose you want to investigate F(X) = SIN(X)/X as X tends to 0. To begin, define:

F(T) = SIN(T)/T,
X(T) = 1/T,
Y(T) = F(X(T)),
T = 0, and
Delta-T = 10.

press -A to generate an array, and press ↓ to scroll the table for increasing positive values of T. Notice that X(T) decreases to 0 and Y(T) tends to 1. If you start at T = 0 and use ↑ to scroll the table through negative values of T, X(T) increases to 0, and again Y(T) tends to 1.

Press Esc to end this example.

Parametric Intersection Points

The next example uses both the Array and Graphics modes to investigate a parametric graph. This example involves the integrated application of several commands, so it is more complicated than most of our examples. Hence, it is optional.

Example 61: The object is to investigate the graph of

X(T) = T * (T – 3) * (T – 1.7) + 2

and

Y(T) = T * (T – 2) * (T + 1) – 2.

In particular, we want to find the intercepts and the point where the graph intersects itself.

To begin,

press ⌘-R to restart in Parametric format,

define X(T) and Y(T) as indicated, and

press ⌘-G to plot the graph.

Notice that the graph has X- and Y-intercepts and intersects itself once.

Use ⌘-P, Project/Plot to get an initial estimate of the coordinates of the X-intercept: press ⌘-P, select Lines, and press Return.

Use ← and → to adjust the projections so that Y is as near to zero as possible. The horizontal projection line should be as near to the X-axis as possible. You may be able to adjust the horizontal projection so that it is on the X-axis. (This changes it from white to green on a color monitor.) Notice that T = 2.27 and X(T) = 1.05.

Now, press Esc to exit the Project/Plot routine, and press ⌘-A to generate an Array. The current values of T, X(T), and Y(T) are the same on the Array display as you left them on the Graphics display. The goal now is to adjust Y(T) as near to zero as possible by decreasing Delta-T.

Since the graph crosses the X-axis at the intercept, Y(T) changes algebraic signs at the intercept. Simply search for sign changes in the Y(T) column of the table, and adjust Y(T) to its smallest value; then decrease Delta-T, and repeat the process.

After several repetitions using 1E –3, 1E –4, 1E –5,. . ., 1E –9 for Delta-T, we obtain T = 2.26953084, X(T) = 1.05581908 as the X-intercept, and Y(T) = 0. You may not get exactly 0 for Y(T). To end your search, press Esc to return to the **?** prompt.

The method for approximating the Y-intercept is similar. First, press ⌘-P to project an approximation by adjusting the X-value to as near zero as possible. Start with Delta-T = .1 and reduce it to.01. Adjust the vertical projection line so that it is near or on the Y-axis. Then press Esc, and press ⌘-A to generate an Array. Search for sign changes in the X(T) column as you reduce Delta-T through the values 1E –3, 1E –4,. . ., 1E –9. We will obtain X(T) = 0 and Y(T) = –1.51425644 as the Y-intercept. (You may not get exactly 0 for X(T). To return to the **?** prompt, press Esc.

The technique for finding the point where the graph intersects itself is more involved. We will use ⌘-Z, Zoom to zoom in on the point. The graph passes through the point twice, so we must find two values of T that correspond to the same point. We will store successive approximations to these two values of T in the parameters A and B as we proceed.

To begin, press:

⌘-M to select Graphics and return to the Graphics display,
T to change T = 0.
⌘-T to change Delta-T to 1E –1,
⌘-P to select Lines,
← and → to project the intersection point as nearly as possible (then, T = –.1),
A to select A = T and store T = –.1 in A,
→ to move the projected point around the loop and project the intersection point again,
B to select B = T and store T = 2 in B, and
⌘-T to change Delta-T to 1E –2.

Now we are ready to zoom in on the intersection point. Proceed as follows. Press:

1) ⌘-Z and enter 10 for both X-MAGNIF and Y-MAGNIF,
2) ← and → to sketch a section of the graph for T near B,
3) T to assign T = A,
4) ← and → to sketch a section of the graph for T near A,
5) ⌘-T to decrease Delta-T to 1E –3 and ← or → for a better projection of the intersection point,
6) A to select A = T and save an improved value for A,
7) T to assign T = B,
8) ← and → for an improved projection of the intersection point for T near B, and
9) B to select B = T and store an improved value for B.

Next, we repeat these nine steps again but reduce Delta-T to 1E –4 in step 5. As we continue to repeat the nine steps, we reduce Delta-T successively to 1E –5, 1E –6,. . . , 1E –9 at step 5, obtaining improved values for A and B each time.

Eventually we obtain A = –.116287516, B = 2.03520644, X = 1.34180404, and Y = –1.78252029. You may want to end this example before achieving this degree of accuracy.

The above graphic/numeric technique for finding points at which parametric graphs intersect themselves is more involved than most techniques illustrated in this manual. The example is included to show the integrated use of several features of *Graphics Calculator* to solve a nontrivial problem. You might want to give some thought to the difficulty of solving such problems algebraically, even in those special cases where the simultaneous solution of the resulting system of two equations in two unknowns can be obtained by elementary methods.

Calculating in Parametric Format

The Calculator display is also available in Parametric format. Press ⌘-M, as usual, to change modes. Six of the eight functions of T are displayed at the top, just as on the Array display. The same parameters are available as in Function format. You can also use the variable, T, in calculations, and you can assign values to it. But you can't assign values to X or Y. If you press X or Y, the functions X(T) and Y(T) are presented for modification; nor can you use X or Y as variables to define functions or parameters. Use the functions X(T) and Y(T) instead.

As noted earlier, when you change from Function to Parametric format, *Graphics Calculator* restarts with the default setup. All functions and parameters are automatically redefined as 0, and the independent variable is T.

With the few differences just mentioned, the Calculator mode operates the same way in both Function and Parametric formats.

Polar Graphs

A special case of parametric graphs is polar graphs. The equations of transformation between parametric and polar coordinates are

$$X(T) = R(T) * COS(T)$$

and

$$Y(T) = R(T) * SIN(T)$$

To graph the equation R = R(T) in polar coordinates, press R and define R(T). Also define X(T) and Y(T) as indicated. You will probably want to change the T-interval to [0,2π], the number of points to 101, and Delta-T to 2π/100. (Use ⌘-I, ⌘-N, and ⌘-T, respectively.) An alternative is to load a Polar setup from the main disk. Just press ⌘-S, select LOAD, and load the file named POLAR from the program disk.

If you intend to use such a setup often, press ⌘-S to save it on any auxiliary disk with the filename POLAR. Then, anytime you want to plot polar graphs, just load the setup POLAR from the setup menu of either the program disk or your auxiliary disk.

Example 62: There are many popular polar graphs. We suggest a few here. You may want to try your own examples.

First,

press ⌘-R to restart in Parametric format,
enter or load the Polar setup indicated above,
press R to define R(T), and
press ⌘-G to plot the graph.

Try the following with several values for A, B and E.

R(T) = A * SIN(B * T) ; ROSE,
R(T) = A * SIN(B * T)^2 ; ROSE,
R(T) = B – A * COS(T) ; LIMACON,
R(T) = A * E/(1 – E * COS(T)) ; CONIC SECTION.

Notice how changing B changes the number of petals on the roses. If A = B, the limacon is a cardioid.

You may need to change the coordinate system to get a better graph of conic sections if E > 1. Recall that E is the eccentricity of the conic. For E < 1, the graph is an ellipse, for E = 1 it is a parabola, and for E > 1 it is a hyperbola. If you don't want the asymptotes of the hyperbola plotted, press ⌘-U to select unconnected points.

For the following graphs, press ⌘-I to change the T-interval to [0,20π] and ⌘-N to increase the number of points to 204. Then try the spirals

R(T) = T/A

and

R(T) = A * EXP(–B * T).

Before plotting the graph, erase the previous graphs and values by using ⌘-E.

For the first function, try A = 15. Plot the graph unconnected first, then connected (press ⌘-U). Notice the pinwheel effect. (See Figure 17.) Also, press ⌘-C to try some different colors without erasing the previous graph.

For the second function, try A = 5 and B = .05.

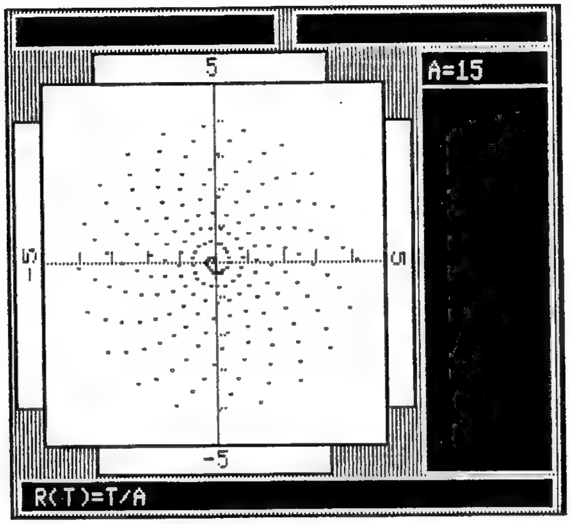

Figure 17.

There are many more interesting parametric and polar graphs not mentioned in our examples. Consult your mathematics handbook for more examples.

You may also use ⌘-P, Project/Plot to plot and project points on polar graphs just as on any other parametric curves. The current values of X, Y, and T are projected. To obtain a value for R(T), just press Esc to exit the projections routine, and assign the value of R(T) to a parameter. The value is printed at the right side of the Graphics display. The full 9-digit representation is available on the Calculator display.

Another way to see R(T) is to assign U = T while using projections, exit projections, assign T = R(T), and press T again. When you press T the second time, its full 9-digit current value, R(T), is displayed. If you enter U for T before pressing Return, you are ready to resume projections exactly where you left off.

Programming ***Graphics Calculator***

Graphics Calculator is a programmable calculator: you can record and save sequences of keystrokes representing various operations and then have this sequence played back later; each operation will be performed as if you were keying it in "live." In this section, then, you will learn how to program *Graphics Calculator,* making use of what you have already learned by using *Graphics Calculator* interactively.

Automatic Operation Sequences

We refer to a sequence of characters used for commands and definitions as an Operation Sequence (abbreviated O-Sequence). Using ♂-O, O-Sequence, you can save O-Sequences as they are typed and save them as files on an auxiliary disk. Later, using ♂-O again, you can load and run the O-Sequence in the same order as originally typed. *Graphics Calculator* behaves exactly as if the O-Sequence were coming from the keyboard, except that entry is much faster.

All commands are available in an O-Sequence, including ♂-S, Setup and the ♂-O command itself. You can use an O-Sequence to load a setup automatically, modify it with individual commands or definitions, and chain to another O-Sequence. Options are also available which allow you to loop through (repeat) an O-Sequence a specified number of times, or while a specified condition is satisfied. For specific applications, you can design O-Sequences to implement algorithms that you use repeatedly.

Demonstrations

The ♂-O, O-Sequence command allows you to prepare the details of a demonstration in advance. To improve demonstrations, you can insert appropriate pauses in an O-Sequence, and have text messages appear at the bottom of the display. Then, at presentation time, you can step through the demonstration simply by pressing the space bar at each designated pause. For example, the demonstrations offered on the Demonstration Disk were created using the ♂-O, O-Sequence command. In Example 64 you will see how to run these demonstrations using ♂-O, Run.

If you press Esc to interrupt an O-Sequence, you are returned to keyboard control with the current setup (as determined by the O-Sequence) intact. You can proceed from that point interactively. On computers with 128K of memory, you can resume the O-Sequence exactly where you pressed Esc. This means that you can interrupt a demonstration, experiment interactively, and resume just where you left off.

The setup is retained when an O-Sequence ends. This lets you set up automatically, run a demonstration, and then turn control over to a user at the keyboard to continue interactively.

Example 63: This example familiarizes you with the ⌘-O, O-Sequence menus and the operation of some of the selections.

Be sure the main *Graphics Calculator* disk is in drive number 1, and that ⌘-D, Disk Drive in Use, is set on 1. To begin, press ⌘-O. The menu

O-SEQUENCE: [RUN] RESUME MAKE CAT DEL

appears at the bottom of the display. Briefly, these selections are as follows:

RUN	is used to run an O-Sequence
RESUME	is for resuming the last O-Sequence at the point where Esc was pressed while it was running.
MAKE	begins the procedure for entering an O-Sequence.
CAT	displays the catalog of O-Sequence filenames on the current disk.
DEL	is used to delete O-Sequence from the current disk.

The SAVE selection doesn't appear on this first O-Sequence menu because there is nothing to save yet. If ⌘-O is pressed at some point *after* you have selected MAKE on this first menu, the following menu appears:

O-SEQUENCE: [CHAIN] SAVE EXIT

Then you can insert a chain at the end of the O-Sequence you have just entered, save it, or exit the Make mode without saving.

To practice with the ⌘-O, O-Sequence command, insert the *Graphics Calculator* Demonstration Disk into drive 1. Try the CAT selection first. (If you are not at the first O-Sequence menu, press Esc to get to the **?** prompt and ⌘-O to get the menu.) Press ← and → to select CAT. When you press Return, a catalog of O-Sequence filenames appears. These are the filenames of the O-Sequences saved on the Demonstration Disk. You can use the files D1, D2, D3, D4, and D5. (Other O-Sequence filenames in the catalog are also used in demonstrations, but they require special setups to run properly. You should not try to run them.)

Try the selection RUN. When you press Return, you are prompted to enter the filename. Type D1. When you press Return, the demonstration begins. It is a demonstration of loading setups that also shows how to use *Graphics Calculator* to analyze quadratic functions.

During the demonstration, respond to the prompt (@ or PRESS SPACE BAR TO CONTINUE) by pressing the space bar when you are ready to go on. If you don't want to see the entire demonstration, hold down Esc until you are returned to the familiar **?** prompt, then proceed interactively with the setup that was current when you pressed Esc. If you want to start the O-Sequence again, press ⌘-O and select RESUME. If your computer has at least 128K of memory, the O-Sequence will start again where you pressed Esc; otherwise, the request is ignored. To use the usual Help screens, you need to press ⌘-R to restart with the default setup first. Then insert the main *Graphics Calculator* disk into drive 1.

Now we will examine each ⌘-O selection in detail.

Make an O-Sequence

The MAKE selection begins the procedure for entering and saving an O-Sequence on an auxiliary disk. Here is a simple example.

Example 64: Before you begin, be sure you have an auxiliary disk in the current disk drive. (See "Making Auxiliary Disks" in the chapter *Loading and Saving Setups, ☌-S.*)

Suppose you want to demonstrate the effect of changing M on the graph of F(X) = M * X + B. A reasonable plan is to define F, initialize M and B, and plot the graphs as you change M. To begin entering the O-Sequence, press ☌-O, select MAKE, and press Return.

The **?** prompt is replaced by the @ prompt to indicate that you are using the ☌-O, O-Sequence command. Proceed as if you were in the interactive mode. All commands and definitions have the same effect as before, but now every character used is recorded in your O-Sequence.

It is a good idea to initialize the setup first, then reduce the number of points plotted so that the lines will be graphed quickly. Press

☌-R to restart with the default setup,
☌-N to change to three points,
F to define F(X) = M * X + B,
M to define M = −2,
B to define B = 1,
☌-G to graph F,
M to define M = M + .5 (to increment M), and
☌-G to graph F.

Repeat the last two steps seven more times—i.e., repeat the following steps until M = 2.

Press M and Return to increment M.
Press ☌-G to graph F.

Now, if you have successfully entered the indicated O-Sequence, you are ready to SAVE it. If not, you will want to start over. In either case, press ☌-O.

The following menu appears:

O-SEQUENCE: [CHAIN] SAVE EXIT

If you don't want to save this O-Sequence, select EXIT. When you press Return, your O-Sequence is erased from memory, and you exit the ☌-O, Make routine. You cannot press Esc to exit the ☌-O, Make routine, because Esc will be recorded as part of your O-Sequence.

If you are ready to save your O-Sequence, select SAVE from the above menu. Next, you are prompted to enter the SAVE filename; enter SLOPE. Be sure you have an auxiliary disk is in the current disk drive before pressing Return. (If you accidentally get a disk error, insert the auxiliary disk, press the space bar as directed, and re-enter the filename. Your O-Sequence is still in memory, so you can still save it.)

RUN an O-Sequence

Now, assuming you have successfully saved the O-Sequence, SLOPE, you can RUN it. Proceed as follows:

Press ♂-O, select CAT, and press Return.

Notice that SLOPE is now an O-Sequence filename.

Select RUN, enter filename SLOPE, and press Return.

Your O-Sequence is retrieved from the disk and repeated as you originally typed it. But unless you are a fast typist, the repeat performance is quite a bit faster than the original!

Example 65 improves the O-Sequence you have just run. For most purposes, it runs too fast. Pausing between graphs would help. For some, it may be too noisy. All of the sound prompts are not needed. And a few subtitles on the displays would help us to interpret them.

Pauses in O-Sequences

To produce a pause in an O-Sequence, press the @ key. At almost any time while you are making an O-Sequence, pressing the @ key produces a distinctive sound to let you know a pause has been inserted in the O-Sequence. When you RUN the O-Sequence and a pause is detected, all action stops, and the @ prompt is displayed at the lower right of the screen (just as the ? is usually displayed.) When you are ready to continue the O-Sequence, press the space bar. The sequence continues until another pause is encountered. An O-Sequence also pauses at each PRESS SPACE BAR TO CONTINUE message, so you don't need to press @ to insert these pauses.

Quiet Operation

To avoid excessive sound, press ♂-Q for Quiet Operation. The simplest procedure, if you do not use ♂-R in the O-SEQUENCE to restart, is not to use ♂-Q while making an O-Sequence, but to activate ♂-Q just before you run the O-Sequence. (Pressing ♂-Q to turn the sound off as you make the O-Sequence would leave you without sound while *making* the O-Sequence as well.) When the run is complete, you can turn the sound back on again by pressing ♂-Q. However, if your O-SEQUENCE uses ♂-R to restart (as ours does), the sound is toggled on. You must turn it off again in the O-SEQUENCE. This difficulty is avoided if you load a setup rather than using ♂-R to restart.

Subtitles and Messages

For subtitles or messages at the bottom of displays, press the **Z** key while making the O-Sequence. If you press Z at the @ prompt, the input cursor appears at the lower left of the display. Now you can type up to 34 characters. You may use any of the characters in Appendix 1 for Titles and Labels; other characters will be ignored.

When you press Return, the subtitle remains until another key (except @) is pressed. You should generally follow the Return by an @ to produce a pause, so that there is time to read the message during a run. Several successive lines may be displayed by repeatedly pressing Z, typing the message, pressing Return, and pressing @.

Now we will use the above information to improve the O-Sequence of the last example. (We will leave the sound on.)

Example 65: Proceed as follows:

Press ⍝-O, select MAKE, and press Return.
Press ⍝-R to restart with the default setup.
Press ⍝-N to change to 3 points.
Define F(X) = M * X + B ;LINEAR FUNCTION.
Press @ to insert a pause.
Define M = –2 ;SLOPE.
Press @ to insert a pause.
Define B = 1; Y-INTERCEPT.
Press @ to insert a pause.

The pauses allow time for observers to read the definitions, for discussion, or for use of other media at RUN time. Notice that functions can be labeled as usual. Now press:

⍝-G to graph F
Z, type a subtitle (such as NEGATIVE SLOPE or GRAPH FALLS),
@ to pause for reading the subtitle,
M to define M = M + .5; SLOPE,
⍝-G to graph F,
Z and type a subtitle commenting on the graph, and
@ to insert a pause.

Repeat the last 4 steps until M = 3. Of course, the subtitles or comments depend upon which observations you wish to emphasize. You may want to point out that the graph rotates as the slope increases, that it is horizontal when M = 0, or that it rises when M > 0. You can skip some of the subtitles and pauses—or, if necessary, you can use additional comments.

When the last graph has been plotted and the last comment entered, it is time to save the O-Sequence. At the @ prompt,

press ⍝-O, select SAVE, and press Return.
Enter SLOPE for SAVE filename, and press Return.

The previous SLOPE file on the auxiliary disk is replaced.

When the **?** prompt returns, you can run the O-Sequence:

Press ⍝-Q select Yes to turn the sound off, and press Return.
Press ⍝-O, select RUN, and press Return.
Enter SLOPE for RUN filename, and press Return.

Now you have only to press the space bar each time the flashing @ prompt appears. The pauses you inserted will allow you to control the pace of the presentation as it is running. The subtitles emphasize important observations you want to make. You may interrupt the O-Sequence at any time by pressing Esc. The ? prompt will appear, and you can proceed interactively. On machines with at least 128K of RAM, you can press ⍝-O and select RESUME to continue the O-Sequence exactly where you interrupted it.

Remember that the setup used in your O-Sequence is still available for interactive use when the ? prompt returns. For example, presentations using O-Sequences may be

followed by interactive question-and-answer sessions using the same setup. As just described, you can also interrupt the presentation and proceed interactively. Then select RESUME again.

It is a good idea to keep things simple at first. It is easier to make several short O-Sequences than one long involved one. Use of subtitles and pauses improves with practice, so you may want to make some O-Sequences over. This is easier to manage with short O-Sequences. More involved operations can be accomplished by chaining one short O-Sequence to another. It is also possible to edit O-Sequences using a word processor and a File Utility program on the *Graphics Calculator* main disk. See *Editing O-Sequences* for more information.

Maximum O-Sequence Size

The maximum length of a single O-Sequence is 4000 keyboard characters. Only characters pressed at the keyboard count. After 3950 characters, each new character you enter produces a sound signal to warn you that you are approaching the maximum 4000 characters. (The sound must be on.) So you have 50 characters to finish and save your O-Sequence after the sound signals begin.

If you exceed the 4000-character limit, you have no characters left with which to save your O-Sequence, and it is lost. The ♂-O routine is exited automatically if this happens.

Projections and Tables of Values in O-Sequences

The arrows keys ←, →, ↑ and ↓ are counted as characters in O-Sequences, so it is not a good idea to use ♂-P, Project/Plot or ♂-A, Array extensively while making one. Holding down ← or → generates characters quickly, especially with ♂-P, Project/Plot. If projections or tables of values are needed extensively, plan to use them interactively rather than in an O-Sequence. If you use them briefly in an O-Sequence, you must, while making the O-Sequence, insert @ wherever you want the projections or scrolling to pause during the run.

O-Sequence Errors

If an error message occurs during an O-Sequence run, the run halts and exits at the error message. For example, if division by zero occurs in the parameter definition C = A/B, the message

UNDEFINED VALUE IN ENTRY:

A/B

CURRENT VALUE NOT UPDATED

PRESS SPACE BAR TO CONTINUE

appears, just as in interactive use. When you press the space bar to continue, you are no longer under O-Sequence control, and the ? prompt returns. You may proceed interactively with the setup that generated the error. The first thing you may need to do is to correct the error.

It is possible that errors will occur while you are running an O-Sequence, *even though you encountered none while making it.* This can happen when your initial setup during the

running of a sequence differs from your setup when *making* the sequence. Saving an O-Sequence does not save the corresponding setup.

For example, if B = 1 while you are making an O-Sequence, but B = 0 while running it, the assignment C = A/B generates an error during the run. To avoid such occurrences, press ⌘-R to restart and/or ⌘-S to load a setup near the beginning of an O-Sequence as you make it. Another way is to use one O-Sequence for the setup and initialization of a procedure, then to chain to a second O-Sequence to complete the procedure. This is particularly useful if one procedure is used on many different setups. The second O-Sequence is used repeatedly. The setup can also be done interactively before running an O-Sequence, if that is appropriate.

Iteration with O-Sequences

For iteration procedures, it is useful to repeat an O-Sequence a specified number of times, or until a given condition is satisfied. You can loop by specifying the number of repetitions, or you can use a WHILE condition. For more complicated procedures, it is useful to chain from one O-Sequence to another. First, we present an example of loops.

Loops

To cause an O-Sequence to repeat or loop a specified number of times, you must append an integer between 2 and 999 to the filename when you run it.

Example 66: The object is to approximate a zero of a function by the secant method. First, define a function and plot its graph. Press

⌘-R to restart,
F to define F(X) = X * X – 3,
⌘-G to graph F, and
⌘-P to project the right-most zero of F.

When you have adjusted the X-projection to approximately X = 1.75, press

B and assign B = X,
→ once to increment X,
A to assign A = X, and
Esc to exit the Project/Plot routine, and
⌘-M to change to Calculator mode.

Now the current values of A and B are 2 and 1.75, respectively. These are preliminary estimates for the zero of F(X). (The current expressions for A and B are not changed, so they remain at zero.)

You are ready to use ⌘-O, O-Sequence to make and then run an iteration procedure. The first iteration is accomplished as you make the O-Sequence. Press

⌘-O to select MAKE
C to define C = B – (A – B)/(F(A) – F(B)) * F(B),
A to define A = B,
B to define B = C,
⌘-O to select SAVE, and enter SECANT for SAVE filename.

Be sure you have an auxiliary disk in the current disk drive before you press Return to enter the SAVE filename. The short O-Sequence you just made is saved on this disk.

Now, to iterate, enter an O-Sequence filename followed by a colon and an integer constant—say, :3. The corresponding O-Sequence is repeated 3 times, as if it were in a BASIC FOR-NEXT loop. For the current example,

press ♂-O,
select RUN, and
enter SECANT:3 for RUN filename.

The O-Sequence SECANT runs 3 times. You can observe as the intermediate values of A, B, and C are updated 3 times. Then the ? prompt returns.

To check that C is now a good estimate of a zero for F(X), assign E = F(C). The current value of E should be very near, or equal to, zero.

You can use the O-Sequence SECANT for any function F(X). Just provide reasonable first estimates, A and B, for the zeros (with A≠B). Here is another example:

Example 67: This time you do not need to make an O-Sequence. Press

♂-R to restart,
F to define F(X) = X^3 – 4 * X – 2,
♂-G to graph F,
♂-P to approximate the middle zero of F near –.5,
B to assign B = X,
← once to increment X
A to assign A = X, and
Esc to exit the Project/Plot routine.

Now you have two preliminary estimates, A = –.75 and B = –.5, for a zero of F.

Press ♂-M to switch to Calculator mode,

Press ♂-O and select RUN.

Enter SECANT:5 for RUN filename.

This time the O-Sequence SECANT is repeated 5 times. If you check by assigning E = F(C), you will find that the final value of C is a good estimate for a zero of F.

WHILE Conditions

If you enter an O-Sequence filename followed by ":W", you are prompted to enter a WHILE condition. The condition is a logical expression, and the O-Sequence is repeated until the condition is false (has value 0). To illustrate, we use the O-Sequence SECANT from Example 66 again.

Example 68: Suppose you want to repeat the Secant method until the difference in consecutive estimates, A and B, is less than 1E –8. First, set up for the Secant routine. Press

Ó-R to restart,

F to define F(X) = EXP(X) – (X + 2),

Ó-G to graph F,

Ó-P to project a zero of F,

B to assign B = X

→ once to increment X,

A to assign A = X, and

Esc to exit Project/Plot.

Now the current values of A and B are preliminary estimates of the zero, and you are ready for SECANT. Press

Ó-O to select RUN.

Enter SECANT:W for the RUN filename.

Enter ABS(A – B) > = 1E –8 for the WHILE condition.

The O-Sequence SECANT is repeated until the WHILE condition,

$$ABS(A - B) \geq 1E{-8}$$

is false.

Each time SECANT is completed, the condition is checked. If it is still true, the O-Sequence is repeated. If it is false, the O-Sequence is not repeated.

After several repetitions, the **?** prompt returns. Check E = F(C), and note that, since E is close to zero, C is a good estimate of the zero. If C is not as good an estimate as you like, you may repeat the iteration interactively simply by pressing C, Return, A, Return, B, Return, in this order, several times. Or you could run SECANT again with a different WHILE condition.

The logical expression for the WHILE condition can contain any valid BASIC logical expression. It can include functions, parameters, and variables. If you define a counter—N = N + 1, for instance—in your O-Sequence and initialize N = 0 before the run, a condition like (ABS(A – B) > 1E –8) AND (N < 10) causes termination when either N ≥ 10 or ABS(A – B) ≤ 1E –8 is true. This prevents the O-Sequence from repeating more than 10 times. It always runs at least once, because the condition is checked at the end of the O-Sequence. (Some systems check WHILE conditions at the top, so be sure to note the difference.)

Chaining O-Sequences

If ♂-O is pressed while you are making an O-Sequence, the following menu appears:

O-SEQUENCE: [CHAIN] SAVE EXIT

Using ♂-O, Chain while making an O-Sequence signifies that (at run time) you want to continue with another O-Sequence at the end of the current O-Sequence. You will be prompted to enter the name of the next O-Sequence as the chain filename. This name becomes part of the current O-Sequence. Next, the current O-Sequence will be terminated and saved. So you need to supply a filename for the current O-Sequence. Later, when you run the current O-Sequence and the chain filename is encountered at its end, that second O-Sequence is run automatically. Here is an example, using the secant method again.

Example 69: Iteration procedures can be done faster if each expression is not "retyped" each time. When iterating interactively at the keyboard, you don't retype each expression. For example, you just type C, Return to update the current value for C, not the entire expression C = B – (A – B)/(F(A) – F(B)) * F(B). You should do the same for iteration with O-Sequences. This example will show you how.

The setup steps are much the same as in the last example. They are not part of the O-Sequence. Press

♂-R to restart,

F to define F(X) = EXP(X) – (X + 3)

♂-G to graph F,

♂-P to PROJECT the zero near –3,

B to assign B = –3,

→ once to increment X,

A to assign A = –2.75, and

Esc to exit the Project/Plot routine.

Now you have two preliminary estimates for the zero, just as in the previous example. Press

♂-O to select MAKE,

♂-M to select CALCULATOR,

C to define C = B – (A – B)/(F(A) – F(B)) * F(B),

A to define A = B, and

B to define B = C.

Now the initial iteration is done, and there is no need to "retype" C each time. So, press

♂-O to select CHAIN and enter

ITERATE:W for chain filename,

ABS(A – B) > = .5 * 10^(–N) for the WHILE condition, and

SECANT for the SAVE filename.

The current value of N in the WHILE condition is assigned prior to running SECANT and determines the precision of the calculation.

Now your first O-Sequence, "Secant," is saved, and you can return to interactive mode at the **?** prompt. Note that the O-Sequence ITERATE is not chained to during the ⌘-O, Make routine. In fact, you have not even made ITERATE yet. To make "Iterate," press

⌘-O to select MAKE,

C, Return, A, Return, B, Return (in this order),

⌘-O to select SAVE, and

enter ITERATE for SAVE filename.

Now, assign N = 9 and RUN SECANT. Press

N to enter 9

⌘-O to select RUN, and

enter SECANT for the RUN filename.

The O-Sequence SECANT sets up the iteration by "typing" the expressions for A, B, and C. Then it chains to ITERATE with the WHILE condition

ABS(A – B) > = .5 * 10^(–N).

The O-Sequence ITERATE repeats the iteration until A and B agree to N decimal places, without retyping the expressions for A, B, and C. This saves time, especially with long expressions like the one for C.

Now that the O-Sequences of the last example are saved, subsequent use is very efficient.

Example 70: Try the O-Sequences of Example 69 once more to find the right-hand zero of F(X). As in that example,

press ⌘-P to get initial A- and B-values.

Enter the number of decimal places, N = 8.

Press ⌘-O to run SECANT.

The rest should be automatic. When finished, the current value of C is an 8-decimal-place approximation to the zero.

If you apply the O-Sequence SECANT to approximate a multiple zero, the approximations may converge very slowly. For example, if F(X) = (X – π)^3, π is a triple zero, and the SECANT method does not work very well. You should use the Project/Plot command and press ⌘-Z to zoom on such zeros. (See Example 41.)

As mentioned earlier, you can put a counter of the form M = M + 1 in ITERATE and change the WHILE condition to ABS (A – B) > .5 * 10^(–N) AND M < 10. Then the method terminates in 10 or fewer steps. An alternative is to watch the values of A and B, and when you want to terminate the iteration, hold down the Esc key. As soon as the Esc is detected, you exit ⌘-O, RUN, and the ? prompt returns with the setup as generated by the O-Sequences.

Chaining with WHILE Conditions and Loops

You can chain as many O-Sequences as you like. Of course, they must all be on disk(s) in the current drive(s). You can use ♁-D, Disk Drive in an O-Sequence to switch disk drives before chaining, if necessary, or a Pause message to insert another disk.

WHILE conditions and loops take precedence over chains. If an O-Sequence is running with a WHILE condition, or repeating N times, chaining does not occur until the WHILE condition is no longer satisfied, or until N repetitions are complete. After the WHILE condition or loop is terminated, the final ♁-O, CHAIN in that O-Sequence (if any) is executed to chain to the next O-Sequence. When the next O-Sequence is loaded from the disk, the previous O-Sequence is replaced, and previous conditions no longer apply. However, as Example 69 shows, you can impose a new WHILE condition on the new O-Sequence as you chain.

Ensuring that a Setup is Appropriate

As mentioned earlier, it is a good idea, when possible, to restart or load an appropriate setup early in an O-Sequence. The operation of an O-Sequence depends on the setup with which it starts. But there are times when the setup from the calling O-Sequence is the setup you want to pass to the next O-Sequence. For example, you might want to pass preliminary estimates of zeros of a function to the next O-Sequence for iteration. In this case, you don't want to restart, because the preliminary estimates (and the functions from which they were obtained) would be erased.

When you use an O-Sequence that does not establish its own setup, you should be sure that any O-Sequence that begins the chain to it does establish the appropriate setup before chaining.

The File Utility

The File Utility (choice 3 of the *Graphics Calculator* menu) can copy and modify three kinds of files. O-Sequence files (abbreviated O-files) and setup files (abbreviated S-files) can be copied from one auxiliary disk to another or deleted; the corresponding auxiliary disk catalogs are updated in the process. The T-files described below can also be copied and deleted.

You can prepare O-files for editing by converting them to text files (T-files). They are converted to DOS 3.3 text files and saved, so you can edit them from a word processor. This lets you use all of your word processor's power to handle the editing chores. After editing, you can convert them back to O-files for use on *Graphics Calculator*. Separate T-files and O-files and corresponding catalogs are maintained on each auxiliary disk as you proceed. You can maintain a library of your favorite O-Sequence subroutines in text-file form and use your word processor to edit them. In the process, you can save files under new filenames in several similar versions without remaking the entire O-Sequence. When you are done editing, convert them back to O-files for use with *Graphics Calculator*.

Whenever you run the File Utility, you make a choice from the first of the menus below. Use the arrow keys, ← and →, just as with other *Graphics Calculator* menus. EXIT returns you to the *Graphics Calculator* menu. With any other choice, you choose a file type from the second menu and a source drive from the third. If you choose CONVERT, the S-file choice does not appear. If you choose CONVERT or COPY, you must also choose a destination drive from the fourth menu.

UTILITY: [CONVERT] COPY CAT DELETE EXIT

FILE TYPE: [O-FILE] T-FILE S-FILE

SOURCE DRIVE: [1] 2

DESTINATION DRIVE: [1] 2

After the disk drive selections are made, the source drive catalog for the appropriate file type is presented, and—unless you selected CAT (catalog) from the first menu—you are prompted to enter the name of the source file:

SOURCE FILENAME: [ ]

Type the name of the file you want to convert, copy, or delete as it appears on the catalog, and press Return. As a reminder of what you are about to do to a file, the word CONVERT, COPY, or DELETE replaces SOURCE on the SOURCE FILENAME menu above. Then (if you previously selected CONVERT or COPY) the destination drive catalog of the appropriate type is presented, and you are prompted to enter the filename under which you wish to save the file:

SAVE FILENAME: SAMPLE [ ]

Your previous source filename is presented for editing. If that is the name you want, press Return. Otherwise, edit the name before pressing Return. The file is saved on the destination auxiliary disk, and the appropriate catalog is updated. If you choose a name already on the destination catalog, the new file will replace the old file.

Finally, you can choose to repeat the same utility, without reselecting the disk drives, by responding to the menu:

ANOTHER: NO [YES]

As usual, filenames should be only eight characters long. You may use the same filename for different file types. For example, if you convert an O-file named SAMPLE to a T-file, you can also use SAMPLE as the name of the T-file; you can also have an S-file with the same name. However, if you save a new O-file called SAMPLE on the same auxiliary disk as another O-file with the same name, the new file will replace the old one.

The File Utility appends suffixes to filenames before saving them on the disk and updating the catalogs. The suffixes do not appear in the File Utility catalogs presented on the screen. The T-file, SAMPLE is actually saved with the filename SAMPLE.TXT. You will need to use this name when you load the file into your word processor for editing. You should also save the edited file from your word processor with a filename containing the suffix .TXT, so that it can be converted back to an O-file. If you save a text file from your word processor with a name (including the suffix ".TXT") that does not appear in the T-file catalog, the catalog is not updated; but you can still convert, copy, or delete that file in the usual way. To get a filename into the catalog, simply use the File Utility to copy the file over itself with the same name.

```
jr)]
n3 ]
jFM*X+B; LINEAR FUNCTION]
@M-2   ;SLOPE]
@B1  ; Y-INTERCEPT]
@g@]
@ZNEGATIVE SLOPE, GRAPH FALLS.]
@ZY-VALUES DECREASE AS X INCREASES.]
@jX-2]
jp@]
@]
@))))@))))@))))@))))@))))@X-2@]
@))))@))))@))))@))))@))))@!ZNOW WE INCREASE THE SLOPE.]
@M-1.5 ;SLOPE]
@g@]
@M-1q]
@g@]
@M-.5]
@g@]
@ZAS M INCREASES, GRAPH ROTATES.]
@M0q]
@g@]
@ZM=0 SO THE GRAPH IS HORIZONTAL.]
@Mq1]
@g@]
@ZM IS POSITIVE SO THE GRAPH RISES.]
@M2]
@g@]
@M3]
@g@]
@ZAS M INCREASES, GRAPH RISES MORE.]
@ZNOW YOU TRY SOME SLOPE VALUES.]
@jo)]
]
```

Figure 18.

Editing O-Sequences

When you begin making longer O-Sequences, you will probably want to edit them after you make them. You can use the file utility CONVERT (described in the preceding section) to convert the O-Sequence to a DOS 3.3 text file (T-file); then you can use your word processor to edit the T-file. Be sure that your word processor is configured to process DOS 3.3 files. When you finish editing, use CONVERT to convert the text file back into an O-file so that you can use it as an O-Sequence on *Graphics Calculator*.

In a T-file produced by CONVERT, all ₢-letter characters in the O-Sequence appear as the corresponding lower-case text letter. For example, if there is an ₢-A in the O-Sequence, a lower-case *a* appears in the text sequence. With few exceptions, all other characters appear unaltered. That is, they appear as the characters on the keys you pressed to generate them in the O-Sequence. Here is a complete list of the exceptions:

Return	appears as	end of line (no character generated)
← and ↑	appear as	{
→ and ↓	appear as	}
Esc	appears as	!

With the above conventions, the text sequence in a T-file appears as a series of lines of text characters which you may edit. Figure 18 shows a T-file obtained from a slightly enhanced version of Example 65.

It takes some practice to learn to "read" a T-file. The Return key is interpreted as a carriage return, so each Return in the O-Sequence terminates a line in the text file. Whether or not Return produces a visible character in the text file depends on your word processor. The Returns in Figure 18 are represented by] at the end of each line. If your word processor does not produce a visible character for Return, two consecutive Returns will produce a blank line. Be careful *not* to interpret the end of a line caused by the "wrap around" of a very long line as a Return.

It is important to note that the last two Returns in Figure 18 are both necessary. Whenever an O-Sequence does not chain to another O-Sequence, these last Returns signify the end of the O-Sequence. If these final Returns are omitted, the O-Sequence will not terminate when you run it.

Only the characters pressed at the keyboard while you are making the O-Sequence (with the above-noted conversions) appear in the T-file. *Graphics Calculator's* responses do not appear. For example, to define the labeled function:

F(X) = M * X + B; LINEAR FUNCTION

you need press only

FM * X + B; LINEAR FUNCTION

and Return. The "(X) = " missing from the expression immediately above is supplied by Graphics *Calculator.* In Figure 18 (and any T-file), you see only the characters you type and the end-of-line caused by Return.

There are many @ characters in Figure 18 because the O-Sequence has many pauses in it. Another character that appears frequently is Z, which initiates the entry of subtitles at the bottom of the screen. Subtitles are usually easy to identify. Remember that subtitles may be used on all displays—including Help screens—at the ? prompt.

You will often see a number of } and { characters in a T-file because the arrow keys are used extensively and for many different purposes. The effect of an arrow key is very dependent on the context in which it is used. When interpreting a converted O-Sequence, you must infer the context from the nearby characters because the display on which the arrow keys were pressed is not before you. The } in the first line of Figure 18 is preceded by **r** and followed by **]**. This means that ₲-R, →, Return was pressed, so the → was used to select YES from the restart menu. On the other hand, the longer sequences of } characters are preceded by **p]]**. This means that ₲-P, Return, Return was pressed to start the Projection routine. Then many arrow keys were pressed to scroll the projections. The intervening @ characters indicate pauses.

The j characters in Figure 18 indicate that ⌘-J was pressed. This toggles printing on and off in the Graphics mode, so that unimportant menus do not distract the viewer when the O-Sequence runs. For more information on ⌘-J and ⌘-K, see Appendix 6.

> Example 71: We have found that an excellent way to interpret and edit a T-file is to print it first, then follow the text sequence as you run the corresponding O-Sequence on *Graphics Calculator*. This helps you to interpret characters in context and to locate changes you want to make. You can practice this now. We have saved the O-Sequence corresponding to Figure 18 on the *Graphics Calculator* disk. It is called "EX.65."
>
> Press ⌘-O and select RUN.
>
> Enter EX.65 for the filename.
>
> Press the space bar to step through the presentation.
>
> As you step through the O-Sequence, try to follow along in Figure 18. You may want to repeat the process several times. Soon you will begin to see the connection between what you see on the screen and the character sequence in the T-file.

Editing O-Sequences is much like computer programming. After you learn the language and get some practice, it becomes much easier. Using the cut-and-paste facilities of a word processor helps. We build most of our long O-Sequences by "pasting" together previously perfected shorter O-Sequences and editing the result.

You can also edit and insert full-screen text displays in your O-Sequence presentations. Appendix 11 introduces this topic.

Part III: Appendices

Appendix 1: Symbols and Keys

Sets of symbols, keys, and characters used by *Graphics Calculator* are listed here. Several subsets used for special purposes are also listed. Detailed definitions, restrictions, and functionalities associated with these characters are contained in other appendices.

Symbol Sets

Alphabet:	**A, B, . . . , Z**
Digits:	**0, 1, . . . , 9**
Arithmetic characters:	**+ - * / . ^) (**
Logical characters:	**= < >** AND OR NOT
Punctuation:	**. : ; ?**
Prompts:	**? @**
Special Character:	π
Command Keys:	**-A, -B, . . . , -Z**
Special Keys:	**Esc** **Return** **Cursor Keys (←, →, ↑ and ↓)** **Space Bar** **Ctrl-Reset** **Z** **Delete** **%** **@**
Characters Not Used:	**! " # $ & ' [ ] { } \ | _**

Symbol Subsets

GENERIC FUNCTIONS

Function format: F, G, H, Q, R, S, with independent variable X.
Parametric format: F, G, H, Q, R, S, X, Y, with independent variable T.

PARAMETERS

A, B, C, D, E, I, J, K, L, M, N, O, P, U, V, W
Here, P = π, and % is used to generate the symbol π on graphics displays.

VARIABLES

X, Y, and T.

In Function format, X is the independent variable, Y is the dependent variable, and T is an ordinary parameter. In Parametric format, T is the independent variable, and X and Y are *functions.*

TITLES AND LABELS

A-Z 0-9 + - * / ^ = < > ( ) % = π = P . ? ; : ,

FILENAMES

A-Z 0-9 - .

Filenames can be no more than 8 characters long.

Appendix 2: Entering Titles, Filenames, and Algebraic Expressions

When you are presented with a blinking square cursor, you may modify existing expressions as well as enter new expressions. The number of characters allowed in an expression is limited and differs for each expression type. For parameters, functions, variable increments, and graphic window bounds, you may enter a BASIC algebraic expression. For filenames, titles, and labels, you may enter text.

Entering Algebraic Expressions

When you enter an expression and press Return, the input routine first checks your syntax. If the expression is incorrect, you are notified immediately (see Appendix 7, *Error Messages*) and given an opportunity to change it. If the syntax is correct, the expression is evaluated (except when defining functions) using current values of each parameter and variable. If evaluation is not possible using these current values, you are notified in a way which depends upon the circumstances. (See "Evaluating Expressions" in Appendix 3.)

For function definitions, the expression is saved as the current defining expression for that function. For parameters and commands, the expression is saved as the current defining *expression,* and its value is saved as the current *value.* For variables, only the current value is saved.

The next time input is required for something which has a current expression, that current expression is presented for modification. For variables, the current value is presented for modification.

If you press Esc rather than Return after changing an entry, the original entry remains unchanged.

Labeling Expressions

If you include a colon or semicolon in an expression, all characters in the expression following the colon or semicolon are treated as a label. This label is ignored during the syntax check and evaluation. The label is presented along with the expression for changing the next time the expression is defined (except for variables). The labels are also displayed on the status displays, where they can be reviewed using the ⌘-S, Setup command.

For example, you can define the function F as

F(X) = M * X + B; LINEAR FUNCTION

and the parameters M and B as

M = 2; SLOPE

and

B = –1; Y-INTERCEPT

Entering Titles

Titles for setups and printed displays can contain the following characters:
A–Z 0–9 + – * / ^ = < > ( ) % = π = P . ? ; : ,

For more information, see *Saving a Setup* or *Printing Graphics Calculator Displays* in Part II.

Entering Filenames

Filenames for setups and O-Sequences can contain 8 characters, including A–Z, 0–9, hyphen (-) and period (.). For more information, see *Loading and Saving Setups, Ó-S,* or *Making an O-Sequence* in Part II.

Editing Entries

Below is a list of keys you can use while making or editing entries:

Key	Command
Return	enters entire expression regardless of cursor position
Esc	escapes without modifying or implementing expression
→	moves cursor right without erasing
←	moves cursor left without erasing
Ó-D	deletes character in current cursor position (same as Delete key)
Ó-I	inserts space at current cursor position for typing new character
Ó-Q	deletes characters, including current cursor position, to end of line
Space Bar	leaves space in an expression and/or erases current position

Note that the commands in the above list have the indicated effect *only* in the input routine (when the square cursor is flashing). Here they act as second-level commands. When the ? prompt is flashing, these commands are first-level and have the effects indicated in Appendix 6. The next example demonstrates each procedure for making and editing entries.

Example 72: First, use ⌘-M to change to Calculator mode. Then press H and define the function $h(x) = x^2 + x - 5$ by typing:

X^2 + X – 5.

Return: When you press Return to enter the definition, H(X) is recorded in the function table immediately as the BASIC expression X^2 + X – 5.

Esc: Now, press H again. When the cursor appears, type XXX, but press Esc rather than Return. The recorded definition of H is left unchanged.

← and →: Press H again. To change X^2 to X^3, use the → key to move the cursor to the right, and type a 3 over the 2. To enter, leave the cursor where it is and press Return. H(X) = X^3 + X – 5 is recorded.

⌘-D: Press H again. When the cursor appears in the entry display, move it to the plus sign (+), then press ⌘-D (or the Delete key) twice to delete + X. When Return is pressed, H(X) = X^3 – 5 is recorded.

⌘-I: To insert + X again, first press H. When the cursor appears, move it to the minus sign (–) and press ⌘-I twice. Two spaces are inserted. Now type + X and press Return. The function H(X) = X^3 + X – 5 is recorded.

⌘-Q: To change to H(X) = X^3, press H and move the cursor to the plus sign (+), then press ⌘-Q. All characters, including the cursor position to the end of the line, are deleted. Pressing Return enters this expression for H.

Space Bar: The space bar is used to leave spaces in an expression. This is useful, for example, to make expressions more readable or to leave space for increasing the number of characters in a coefficient later. The spaces are ignored when an expression is executed. The space bar also serves as an eraser.

Appendix 3: Algebraic Expressions

Graphics Calculator uses BASIC syntax for the algebraic expressions that define functions, parameters, and other expressions for numeric factors. An exception is that FN is not used for functions. For example, use G(X) rather than FN G(X). For composite functions, use F(G(X)), not FN F(FN G(X)).

Expression Components

In an algebraic expression, you may use any of the components listed in the following paragraphs with correct BASIC syntax.

VARIABLES

X, Y, and T

In Function format, the independent variable is X. The dependent variable, Y, is changed by the program, but T is not changed, so it may be assigned a value and may be used in expressions. In Parametric format, the only variable is the independent variable, T.

GENERIC FUNCTIONS

Function format: F(X), G(X), H(X), Q(X), R(X), S(X),
Parametric format: F(T), G(T), H(T), Q(T), R(T), S(T), X(T), Y(T)

When you are using a function in another algebraic expression, you may replace the variable (X for Function format or T for Parametric format) with any valid algebraic expression. (Note: composite functions may be used in algebraic expressions.)

PARAMETERS

A, B, C, D, E, I, J, K, L, M, N, O, P (%), U, V, W

NUMBERS

Floating point (e.g., –32.045) or exponential (e.g., –32.045E +12) form is valid. You may also use integers and fractional expressions (e.g., 3/4).

BASIC BUILT-IN FUNCTIONS

SIN, COS, TAN, ATN, EXP, LOG, ABS, INT, SQR, SGN, RND

Angles are expressed in radians, not in degrees. For further details on each built-in function, see your *Applesoft BASIC Programming Reference Manual* (published by Addison-Wesley, ISBN 0-201-17722-6) and Appendix 4 of this manual.

ARITHMETIC OPERATORS

\+ - * / ^ ( )

LOGICAL RELATIONS

> < >= <= (< > or > <) =

LOGICAL OPERATORS

AND OR NOT

Logical operators and relations are used to form and compare logical expressions. A logical expression has the value 1 if true and 0 if false. For details, see your *Applesoft BASIC Programming Reference Manual* (published by Addison-Wesley, ISBN 0-201-17722-6).

Evaluating Expressions

The value of an expression is computed using the current value of each parameter and variable in the expression definition. The precedence of operations is according to BASIC. The precedence of operations, from highest to lowest, is:

Unary Operators

( )

\+ - NOT

Binary Operators

^

* /

\+ -

> < >= <= (< > or > <) =

AND

OR

Operators of the same precedence are executed from left to right. Note that -X^2 is equivalent to (–X)^2, but 1 – X^2 is equivalent to 1 – (X^2). Also note that A < B < C is interpreted as (A < B) < C, not as ((A < B) AND (B < C)). So, when in doubt, use parentheses to control precedence.

Each time an expression value is called for, the expression is evaluated using the current value of each of the parameters and variables in that expression. If the expression cannot be evaluated, the consequences depend upon the situation.

If the value is for assignment and/or display, you are notified that an undefined or overflow value has been encountered. Then the assignment is not made. If the value is for plotting a graph, the corresponding point is ignored.

If the value is for implementing a command, it is rejected and you are prompted to reenter the expression. For further details, see Appendix 7, *Error Messages*.

Appendix 4: Functions

The generic functions in Function format are F(X), G(X), H(X), Q(X), R(X), and S(X). The independent variable is X. In Parametric format, the independent variable is T, and the generic functions are F(T), G(T), H(T), Q(T), R(T), S(T), X(T), and Y(T). [But only X(T) and Y(T) are graphed in Graphics mode or listed in Array mode.]

Defining Functions

Functions are defined on any display when the flashing ? prompt is present. To define a function, press the corresponding letter. When the flashing square prompt appears, enter a valid algebraic expression, or change the previous expression, and press Return. (See Appendix 3, *Algebraic Expressions*.)

For example, to define $f(x) = x^2$, press F, type X^2, and press Return. Syntax is checked immediately. If incorrect, you have a chance to change the defining expression (see Appendix 7, *Error Messages*.).

To change the definition of a function, press the corresponding letter, and change the last definition. When you press Return, the new expression redefines the function.

Function Values

When a function value is called for, the expression defining the function is evaluated using the current values of all parameters and variables in that expression. If evaluation is not possible, the consequences depend upon the situation. See "Evaluating Expressions" in Appendix 3.

Function Labels

The defining expression for a function may be followed by a label using the conventions for labeling expressions. See "Labeling Expressions" in Appendix 2.

BASIC Built-In Functions

You may use BASIC built-in (intrinsic) functions in any algebraic expression. In particular, you may use them to define any of the eight generic functions F, G, H, Q, R, S, X, and Y.

In the following, X may be replaced by any valid algebraic expression. But remember that the independent variable in these expressions is X in Function format and T in Parametric format.

SIN(X)	sine of X, X in radians
COS(X)	cosine of X, X in radians
TAN(X)	tangent of X, X in radians
ATN(X)	arc tangent of X, value in radians (range: 0, $\pi/2$)
SGN(X)	the algebraic sign of X, –1 if X < 0, 0 if X = 0, 1 if X > 0
ABS(X)	the absolute value of X
SQR(X)	the principal (positive) square root of X for X ≥ 0
EXP(X)	exponential with natural base e
LOG(X)	logarithm for natural base e, X > 0
RND(X)	random number generator, values satisfy 0 ≤ RND(X) < 1

To be precise, RND is not a function. For each negative value of X, RND(X) is the first term in a predetermined sequence of pseudo-random numbers. To generate subsequent terms in that sequence, use only positive X. To repeat the most recent term, use X = 0. Each time a specific negative value of X is reused, the same sequence of pseudo-random numbers is initiated.

The following identities are used to generate values for functions that are not intrinsic to BASIC. A more extensive list is provided in the *Applesoft BASIC Programming Reference Manual* (published by Addison-Wesley, ISBN 0-201-17722-6).

1/COS(X)	secant
1/SIN(X)	cosecant
1/TAN(X)	cotangent
ATN(X/SQR(1 – X * X))	arc sine
–ATN(X/SQR(1 – X * X)) + π/2	arc cosine
ATN(SQR(X * X – 1)) + (SGN(X) –1) * π/2	arc secant
ATN(1/SQR(X * X – 1)) + (SGN(X) –1) * π/2	arc cosecant
–ATN(X) + π/2	arc cotangent

[Note: all arc functions are in the range 0, π/2.]

Function Displays

Function definitions are displayed in seven places: at the top of the Array display, the top of the Calculator display, in the fourth Status display, and—optionally—at the bottom of each of the Help, Graphics, Array, and Calculator displays.

In Function format, the definitions of all six functions — F, G, H, Q, R, and S — are displayed at the top of both the Array and Calculator displays. Each time a function is graphed or redefined, its definition appears at the bottom of the current display.

In Parametric format, the definitions of the six functions F, G, H, R, X, and Y appear at the top of the Array and Calculator displays. The functions S and Q are available but are displayed only when redefined.

On any display, to review the definition of a function without redefining it, press the corresponding key (F, G, H, Q, R, or S in Function format; also X or Y in Parametric format) at the ? prompt. The function definition appears at the bottom of the display. Press Esc to leave the definition unchanged. On the Graphics or Array displays, if you want to see the current values of any parameters used in the definition, press Return rather than Esc. This redefines the function and prints the current values of any parameters used in the definitions on the right side of the display.

All eight function definitions appear on the fourth Status display. At the ? prompt, definitions of all parameters, functions, and command expressions can be reviewed using the ⌘-S, Setup command.

Appendix 5: Parameters and Variables

The parameters A, B, C, D, E, I, J, K, L, M, N, O, P, U, V, and W are available. However, P is reserved for π. In Function format, the variables are X, Y, and T, with X as the independent variable in all function definitions. In Parametric format, T is the independent variable in all function definitions. You may not define the values of X and Y in Parametric format, since they are functions.

Defining Parameters

Each parameter has a *current expression* and a *current value.* The default expressions and values are all 0, except that P has the 10-digit expression 3.141592654, which produces the 9-digit current value 3.14159266. (This number has a 6 for the final digit rather than the 5 that correct rounding would produce. The advantage is that SIN(P) = 0, whereas SIN(3.14159265) ≠ 0.) Defining a parameter changes its current expression and/or its current value. However, the expression P—and therefore its value—cannot be changed.

To define a parameter, press the corresponding letter when the ? prompt is flashing, type a valid algebraic expression (or modify the previous expression), and press Return. (See Appendix 3, *Algebraic Expressions.*)

Syntax is checked immediately. If it is incorrect, you have a chance to change the defining expression. (See Appendix 7, *Error Messages.*) If the syntax is correct, the expression is saved as the current expression for that parameter. If Esc is pressed rather than Return, entry is terminated, and the previous definition is not changed.

For example, if you press A, type (A + F(A))/B, and press Return, (A + F(A))/B becomes the current expression for A. Note that self-reference (defining A in terms of A) is permissible in defining parameters but not functions.

After syntax is checked and found correct, the expression is evaluated using the current values of each parameter and variable in the expression. If evaluation is successful, the value obtained becomes the current value of the parameter. In our example, if the value of (A + F(A))/B can be calculated using the current values of A and B, then this new value is assigned as the current value of A.

If the expression cannot be evaluated, the current value of the parameter is not updated, and you are notified. (See Appendix 7, *Error Messages*..) However, if the syntax is correct but the value is undefined, the current expression is updated but the current value is not.

The current value of a variable, X, Y, or T, may be assigned directly as the current value of a parameter without altering the current expression for that parameter. See ♂-P, Project/Plot in Appendix 6 for the method.

Iteration

To update repeatedly the current value of a parameter—A, for instance—without changing the current expression, press A, Return for each update. To update A and B alternately , press A, Return, B, Return. Each time Return is pressed, the respective current value is updated using the current values of variables and parameters in the defining expression.

Parameter Displays

The current *expressions* for all parameters appear on the first Status display. Use ♂-S, Setup to see this display. Individual current expressions may be viewed at the flashing ? prompt.

Note that P has the expression 3.141592654 in the Setup Status display. However, its displayed value is 3.14159266 in all modes. [Notice that this number ends with a 6 rather than the 5 that correct rounding would produce. The advantage is that SIN(P) = 0, whereas SIN(3.14159265) ≠ 0.]

To review the current expression for a parameter—A, for instance—press A at the ? prompt on any display. The current expression is printed at the bottom of the display. If you don't want to update the current value, press Esc.

The current *values* of all parameters can be reviewed at once on the Calculator display (use ♂-M, Mode to change to the Calculator mode).

If a function is defined on the Graphics or Array display, the current value of each parameter used in the defining expression is printed on the right side of the display. A question mark (?) indicates that the displayed values are truncated to fit the display area. However, full 9-digit precision is used when these values are used in calculations.

Similarly, if a parameter is redefined on the Graphics or Array display, the current value is printed on the right side of the display.

Defining Variables

In Function format, you can assign values to the variables X, Y, or T, just as you do for parameters. Like parameters, these variables will have current values; but they will not have current expressions.

The values of X and Y are also changed by the Project/Plot, Array, and Values Assigned options. The values of X and Y are shown at the top of the Graphic display screen.

You can assign and use values of T elsewhere, as its value is never changed by other options.

In Parametric format, you can assign values only to the variable T. The value of T is changed by the Project/Plot, Values Assigned, and Array options.

Variable Displays

To see the current value of X, Y or T (only T in Parametric format) at the bottom of the screen, press the corresponding key whenever the ? prompt flashes. When the value appears, you can change it as described above or press Esc to leave it unchanged.

The current values of X and Y appear at the top of the Graphics display and change as the Project/Plot and Values Assigned options are used. In Parametric format, T appears at the bottom of the screen. Similarly, the values of X, Y, and T are displayed and change on the Array display.

If you press ♂-V during the Project/Plot option, the values of X and Y (and T in Parametric format) are printed in the table at the right of the Graphics display. The current values of X and Y (and T in Parametric format) also appear at the bottom of the first Status display.

Appendix 6: Special Keys and Commands

Using Special Keys

Graphics Calculator is controlled by certain special keys. Here is a list of the special keys and what they do.

ESC

The Esc key is the universal exit key. Pressing the Esc key terminates the current option and returns you to the point at which the option was initiated. If you press Esc at the ? prompt, you will toggle between the Help screens and either the Graphics, Array, or Calculator display (whichever was used last). If you press Esc several times, you must eventually return to the last-used Help screen.

Some options do not accept the Esc key. For example, it is not accepted while plotting is in progress with the ₲-G, Graph Function command. Nor is it accepted while printing is in progress. To get to the main menu in these cases, press the Esc key when the ? prompt returns. (To exit a running O-Sequence, press Esc at the @ prompt, or hold down the Esc key until Esc is detected and the ? prompt returns. You cannot exit the option for making an O-Sequence with the Esc key. Instead, press ₲-O and select EXIT.)

RETURN

The Return key is pressed to complete an entry. If a selection is made from a menu using the cursor keys, Return sends that selection to the program. Return also enters an expression in the input routine (see Appendix 2).

CURSOR KEYS

The cursor keys are used to scroll in several ways. Often, menus are presented at the bottom of displays with the current selection in reverse video. To change the selection, press ← to move left and → to move right. Similarly, when the input editor is used to enter expressions, ← moves the cursor to the left, and → moves it to the right. In both cases, press Return to complete the selection or entry.

Use the ← and → keys to move back to the previous screen, or ahead to the next screen, in the Help and Status displays. (The space bar and → have the same effect in the Status displays.)

In the Project/Plot option, use ← and → cursor keys to decrease or increase the value of the independent variable (X or T). In the Array mode, use ↑ or ↓ to decrease or increase the value of X or T.

SPACE BAR

The space bar is the continuation key. In many options, you are prompted at the bottom of the screen to PRESS SPACE BAR TO CONTINUE. Also press the space bar to continue at an @ prompt while running an O-Sequence. The space bar is used to move forward through Status displays.

CTRL-RESET

Ctrl-Reset key reboots the system, starting the program over. Use Ctrl-Reset at any time to return to *Graphics Calculator*'s opening menu. (You may also use ₾-R, Restart and select MENU to return to this menu directly.)

THE Z KEY

The Z key is used to display labels or titles at the bottom of displays. It is used primarily while making O-Sequences . If Z is pressed at the ? prompt (or at the @ prompt while making an O-Sequence), the input cursor appears at the lower left of the screen. Up to 34 characters may be typed. Press Return to terminate entry. The message is erased when the next key is pressed (except @). Only the characters listed in Appendix 1 for Titles and Labels may be used. Other characters are ignored.

When the character Z is encountered while running an O-Sequence, the sound is toggled off if it is on (and on if it is off) while the subsequent title is displayed.

THE @ KEY

The @ key is used to insert pauses in O-Sequences. When you are making an O-Sequence, pressing the @ key produces a sound prompt to let you know the key has been detected. When you run the O-Sequence, it pauses at the point where @ was pressed, and an @ prompt appears at the lower right of the screen. To continue running the O-Sequence, press the space bar.

The @ prompt replaces the ? prompt when making and running O-Sequences .

THE % KEY

Press % to produce the π symbol on the graphics display. On text displays, % appears as P, but on graphics displays, it appears as the symbol π. It has the value associated with P, as explained in Appendix 5.

Accessing Commands

This section summarizes the main purpose and operation of each first-level command. All commands keys used at the ? prompt are referred to as first-level command keys. A first-level command is initiated at the ? prompt by pressing the corresponding control character. For example, you would press ₾-A (hold down the ₾-key and press A) at the ? prompt to initiate the Array command.

Entries made while a first-level command is operating are called second-level commands. Second-level input is described in this section for each command that accepts such input. Most first-level commands accept only a limited number of second-level commands.

Note that a given key may have entirely different effects at the first and second levels. For example, at the ? prompt, ₾-F changes between Function and Parametric formats; but with the Project/Plot command, ₾-F changes the current function whose graph is projected.

This appendix is only a brief summary of the purposes and operation of each command. For detailed examples, instructions, and techniques, see the main text of this manual.

♂-A, ARRAY OF VALUES

Purpose
Display scrolling tables of function and variable values for functions or parametric equations.

In Function format, the independent variable is X; tables are generated for X and for the functions F(X), G(X), H(X), R(X), S(X), or Q(X). In Parametric format, the independent variable is T, and tables are generated for the functions X(T) and Y(T).

Operation
On any display in Function format, call the Array command (♂-A) at the ? prompt, use the cursor keys ← and → to select a function F, G, H, Q, R, or S from the menu that appears, and press Return. If not already in Array Mode, the program then switches to Array Mode. A table of values of the the chosen function and the variable X is displayed. The current value of the independent variable, X, and the function value, say F(X), are emphasized by a reverse video cursor. Use the ↑ or ↓ keys to move the cursor to lower or higher values of X, respectively, and to scroll the table. (You may also use ← or →.) The function value, F(X), is assigned automatically as the current value of the variable, Y, as the current value of X changes.

In Parametric format, press ♂-A at the ? prompt on any display, use ← or → to select Yes, and press Return. A table of eleven pairs of values for the parametric functions, X(T) and Y(T), is generated. The current value of the independent variable, T, appears at the bottom of the display and the corresponding values of X(T) and Y(T) are emphasized by a reverse video cursor in the table. Use the ↑ and ↓ keys to move the cursor down or up, respectively, and to scroll the table. The function values, X(T) and Y(T), are assigned as the current values of the variables, X and Y, respectively, as the current values of T changes.

Second-level Input
<u>GENERAL</u>
↑ and ↓: Pressing the ↑ or ↓ cursor keys in the Array option moves the reverse video cursor up or down in the table as the current value of the independent variable, X or T, is incremented accordingly. When the cursor reaches the top or bottom of the table, the table scrolls.

Esc: Press Esc to return to the ? prompt on the Array display. Then all first-level commands are available.

<u>FUNCTION FORMAT</u>
X: Press X, type a valid BASIC expression (see Appendix 2, 3, and 8), and press Return. The value of the expression is assigned as the current value of X, and a new table is generated.

Delta-X: Press ♂-X, type a valid BASIC expression with positive value for the increment, and press Return. A table of values is generated with the new Delta-X and the current value of X at the center. Nonpositive values of Delta-X are rejected, and the prompt to enter Delta-X is repeated. In the Array option, use Delta-X only in Function format.

PARAMETRIC FORMAT

T: Press T, type a valid BASIC expression for T, and press Return. The value of the expression is assigned as the current value of T, and a new table is generated with the corresponding values of X(T) and Y(T) in the center and T at the bottom of the display.

Delta-T: Press ♂-T, type a valid BASIC expression with positive value for the increment, and press Return. A table of values is generated using the new Delta-T and current value of T. Nonpositive values are rejected, and the prompt to enter Delta-T is repeated.

♂-C, COLOR OF GRAPH

Purpose

Change the color (or linestyle, on a monochrome display) of graphs. The colors black, white, green and violet are available in medium resolution. Black is used to erase other colors. High resolution plotting, which uses a thinner linestyle (one pixel wide) is also available.

Operation

Press ♂-C and use the cursor keys ← and → to make a selection from the menu:

COLOR OF GRAPH: BLK GRN VLT [WHT] HRS

Press Return to complete your selection. The next graph you plot is plotted in the selected color or linestyle. See also ♂-U, Unconnect or Connect Points to plot points in the selected color, either unconnected or connected.

♂-D, DISK DRIVE IN USE

Purpose

Change the disk drive in use between drive 1 and drive 2 on two drive systems.

Operation

Press ♂-D and use the cursor keys ← or → to make a selection from the menu:

DISK DRIVE IN USE: [1] 2

After you press Return to complete your entry, the computer reads and writes from the drive you selected until you change your selection, or until you restart using ♂-R. When you restart, drive 1 is selected automatically.

♂-E, ERASE DISPLAYS

Purpose

Erase the graphs and/or tables of values on the Graphics display.

Operation

Press ♂-E and use the cursor keys ← or → to make a selection from the menu:

ERASE DISPLAYS: [GRAPHS] VALUES BOTH

Press Return to complete your selection.

When you select GRAPHS, the coordinate window is erased, and the coordinate axes are re-drawn. The values in the displays above and to the right of coordinate window are not erased. When you select VALUES, the values for X and Y displayed above the coordinate window, and any values in the table to the right of the coordinate window are erased. The current values and/or expressions for variables and parameters are not affected by erasing these tables.

If you select BOTH, then the coordinate window and the tables are both erased.

♂-F, FORMAT

Purpose
Change between Function format and Parametric format of operation.

Operation
Press ♂-F and use the cursor keys ← or → to make a selection from the menu:

FORMAT: PARAMETRIC [FUNCTION]

When you press Return to complete your selection, *Graphics Calculator* restarts in the format you selected with the default setup.

If you select Function format, you can define the six generic functions F, G, H, Q, R, and S, with independent variable X as described in Appendix 4. These functions are available on the Graphics display for graphing, on the Array display for tables of values, and on the Calculator display for calculating.

If you select Parametric format, the independent variable is changed to T in all functions, and two additional functions, X(T) and Y(T), are available. Only the pair X(T) and Y(T) are used for graphs on the Graphics display and tables on the Array display. The other six functions are available for calculations and for use in algebraic expressions in other functions and parameters. Due to limited display space, the functions S(T) and Q(T) do not appear in the function tables at the top of the Array and Calculator displays, but they are available nevertheless.

In Parametric format, you cannot use the variables X and Y. If you press X or Y at the ? prompt, the function X(T) or Y(T), not the variables X or Y, is presented for changing. Nor can the variables X and Y be used to define expressions in Parametric format. You will get an error message if you try.

Second-level Input
All first- and second-level command keys have similar effects in both Function and Parametric format. Any differences are explained in the command descriptions in this appendix.

♂-G, GRAPH FUNCTION

Purpose
Plot the graphs of functions in Function format and parametric equations in Parametric format.

Operation
In Function format, press ♂-G and use the cursor keys ← or → to select a function from the menu:

GRAPH FUNCTION: [F] G H R S Q

When you enter your selection, the graph is plotted on the coordinate window of the Graphics display.

When you press Ꮳ-G in Parametric format, the menu displayed is:

GRAPH X(T) Y(T): NO [YES]

If you select YES, the graph of the parametric equations, X = X(T) and Y = Y(T), is plotted.

Only the parts of a graph inside the coordinate window are plotted. If a calculated point is outside the coordinate window, a small arrow at the edge of the coordinate window points to the general location of the point, and a sound is emitted. If an attempt is made to evaluate a function outside its domain, a "ticking" sound is emitted, and no point is plotted.

Graphing with GRAPH FUNCTION can be interrupted only by pressing Ctrl-Reset, which causes a restart of *Graphics Calculator.*

Ꮳ-H, HELP SCREENS

Purpose

Review Help screens that briefly explain the operation of *Graphics Calculator* and list BASIC built-in functions. Help 2 is an index of all Help screens. You may make any valid entry at the ? prompt on any Help screen.

Operation

Press Ꮳ-H and enter the number, 1 through 15, of the Help screen you want. The Help screen you selected then appears. To move to the next Help screen, press →. To move back to the previous Help screen, press ←. Press Esc to return to the display at which the Help command was initiated.

Auxiliary Disks may contain their own customized Help screens that are different from those on the *Graphics Calculator* program disk.

Ꮳ-I, INTERVAL CHOICE

Purpose

Assign the left and right endpoints of the interval over which the independent variable, X or T, varies during plotting.

Operation

In Function format, press Ꮳ-I. When the prompt "X-INT LEFT" appears, type a valid BASIC algebraic expression for the left endpoint of the X-interval. Press Return to complete your entry. When "X-INT RIGHT" appears, type a valid BASIC algebraic expression for the right endpoint. Press Return to complete this entry. Valid BASIC algebraic expressions are described in Appendix 3.

In Function format, the values of the BASIC expressions entered must satisfy the inequality:

$$\text{LEFT SIDE} \leq \text{X-INT LEFT} < \text{X-INT RIGHT} \leq \text{RIGHT SIDE}$$

where "left side" and "right side" are the left and right sides of the coordinate window, respectively, as described under Ꮳ-W, Window Coordinates. If this inequality is not satisfied, the first entry that violates it is rejected, and the prompt to enter the value reappears.

In Parametric format, the Interval Choice command prompts you to enter "T-INT LEFT" and "T-INT RIGHT," the left and right endpoints of the T-interval. The only restriction here is that the left endpoint is less than the right endpoint. If not, the right endpoint is rejected, and the "T-INT RIGHT" prompt is repeated.

If Esc is pressed before the right endpoint entry is complete, the option is exited, and the interval is not changed. If you have difficulty, press Esc and start over.

Second-level Input

BASIC Expressions: Any valid BASIC algebraic expression using functions and parameters as described in Appendix 3 may be used to assign left and right endpoint values. The endpoint values are subject to the restrictions stated above.

Esc: To end the Interval Choice command before completing it, press Esc. The interval is not changed, and the ? prompt returns.

✪-L, LIST TEXT SCREEN TO PRINTER

Purpose

Print any text screen on a printer. This includes the Array and Calculator screens as well as all Help, Catalog, and Setup screens.

Operation

Your printer control card must be in slot #1. The printer must be on line (sometimes called "select mode"), and it must have paper in it. To print the current text display, press ✪-L. The menu presented is:

```
LIST TEXT SCREEN TO PRINTER:   NO   [YES]
```

Press Return to select YES, or use the cursor keys to select NO, then press Return. If you select YES, the next menu is:

```
ARE YOU SUPPLYING A TITLE?   [NO]   YES
```

Use the cursor keys ← or → to select YES or NO, and press Return. If you select NO, the display is printed.

If you select YES, you may enter a title (up to 34 characters) at the bottom of the display. Use only the characters listed in Appendix 1 as being valid for titles and labels; other characters will not be recognized. When you press Return, the title is centered, and the display is printed. When printing is complete, continue with normal operation.

Second-level Input

TITLES: Text titles may be entered before printing. Use the editing features described in Appendix 2.

Esc: Press Esc to exit the printing option before printing has started. To exit while printing is in progress, you must press Ctrl-Reset to restart *Graphics Calculator.*

♂-M, MODE

Purpose
Switch among Graphics, Array, and Calculator modes.

Operation
Press ♂-M and use the cursor keys ← or → to make a selection from the menu:

MODE:	[GRAPHICS]	ARRAY	CALCULATOR

Press Return to complete your selection. The display changes immediately when you complete your selection of a different mode. The ? prompt is in the lower right corner, indicating that *Graphics Calculator* is awaiting your next entry.

♂-N, NUMBER OF POINTS

Purpose
Change the number of points that are calculated and plotted by the ♂-G, Graph Function option.

Operation
Press ♂-N, and type a valid BASIC algebraic expression when the prompt "NUM POINTS" appears. Press Return to complete your entry. The value of your expression must be in the range 2–1000.

Second-level Input
BASIC Expressions: Any valid BASIC algebraic expression with a value between 2 and 1000 may be entered to assign the number of points. See Appendix 3.

Esc: If Esc is pressed before Return, the Number of Points command is exited, and the number of points is not changed.

♂-O, OPERATION SEQUENCE

Purpose
An Operation Sequence (abbreviated as "O-Sequence") is a sequence of keyboard characters that is recorded in memory as it is typed. The O-Sequence is then saved on an Auxiliary Disk in an Operation File. Later, you can send the O-Sequence back to the computer. The sequence of characters is interpreted just as if it were being typed at the keyboard, only entry is much faster. Appropriate pauses are possible.

Operation
When you press ♂-O at the ? prompt, the following menu is presented:

O-SEQUENCE:	[RUN]	RESUME	MAKE	CAT	DEL

Use the cursor keys ← or → to make a selection, and press Return to complete it.

RUN: If you select "RUN," you are prompted to enter "RUN FILE NAME." Type the name of an Operation File that appears on the catalog of the current Auxiliary Disk. When you press Return to complete the entry, the file is loaded from the disk and interpreted as if it were being typed at the keyboard. The @ prompt replaces the ? prompt.

Each time an @ prompt appears at the lower right-hand corner of the screen, operation pauses. Press the space bar when you are ready to continue. Operation also pauses when a PRESS SPACE BAR TO CONTINUE message appears at the bottom of the display.

At the end of an O-Sequence, the ? prompt returns.

To exit an O-Sequence before it is finished, press Esc at the @ prompt, or hold down the Esc key until the Esc key is detected. It will exit automatically if an error is encountered during a run.

Loops: If you want a run to be repeated a specified number of times, append a colon and a loop index to the RUN FILE NAME. For example, if you enter "TEST:5" for the RUN FILE NAME, the O-Sequence with filename "TEST" is repeated 5 times. The loop index must be a positive integer constant between 2 and 999. Do not use parameters or variables.

WHILE: If you append ":W" to a RUN FILE NAME, you are prompted to enter a WHILE condition. The WHILE condition is a valid BASIC logical expression (see Appendix 3). After you press Return to complete the entry, the O-Sequence repeats until the logical expression is false (has value 0). The condition is checked at the end, so it always runs at least once.

RESUME: On machines with at least 128K of memory, you may resume an O-Sequence after you have pressed Esc to exit a run. This means you can interrupt an O-Sequence at any time, perform interactive operations with the current setup and menu system, and begin the O-Sequence again exactly where you left off.

To resume, press Ó-O, select RESUME, and press Return. If no O-Sequence has been run since the last RESTART, or if you are not using a 128K machine, the RESUME selection is ignored, and the ? prompt returns.

MAKE: If you select MAKE from the O-Sequence menu, the ? prompt changes to @. Now *Graphics Calculator* appears to operate as usual. However, every key you press (except Reset or Ó-O) is recorded in memory. Up to 4000 characters may be recorded. After 3950 characters, a sound prompt warns you that the 4000 limit is near and you must save the O-Sequence.

To quit making an O-Sequence, press Ó-O again. The menu:

```
O-SEQUENCE:   [CHAIN]   SAVE   EXIT
```

appears. Use the cursor keys ← or → to make a selection, and press Return to complete it.

SAVE: If you select SAVE, you are prompted to enter a SAVE FILE NAME. Use only the characters 0-9, A-Z, . , and - for filenames. Filenames may be no more than 8 characters long. Before pressing Return to save the Operation file, be sure to insert an Auxiliary Disk in the current drive.

CHAIN: If you select CHAIN, you are first prompted to enter a CHAIN FILE NAME. This is the filename of another O-Sequence to which want the current O-Sequence (the one you have just completed and are about to save) to chain automatically at run time.

You may append a LOOP index or a ":W" (for a WHILE condition) to a CHAIN FILE NAME, just as described for RUN FILE NAMES. At run time, the O-Sequence to which you chain is repeated the specified number of times, or until the WHILE condition is satisfied. A CHAIN is not executed, however, until any repetition or WHILE condition on the current O-Sequence is satisfied.

EXIT: If you select EXIT at the

O-SEQUENCE: CHAIN SAVE [EXIT]

menu, the MAKE option is terminated, and the current O-Sequence is erased from memory. It is not saved. You cannot use Esc to exit the MAKE option, because Esc may be saved as part of the O-Sequence you are making.

CAT: If you select CAT from the

O-SEQUENCE: RUN RESUME MAKE [CAT] DEL

menu, the catalog of Operation filenames on the disk in the current disk drive is displayed with the same menu at the bottom, so that you can make a selection while looking at the catalog.

DEL: If DEL is selected, you are prompted to enter NAME OF FILE. Type the filename of the O-Sequence you want deleted from the current disk, and press Return. The file is deleted, and you are returned either to the ? prompt or the catalog display. Press Esc to exit the catalog display.

Second-level Input

Space Bar: Use the space bar to continue at the @ prompt during an O-Sequence run and also at a PRESS SPACE BAR TO CONTINUE prompt.

Esc: Press Esc to terminate an O-Sequence run before it ends automatically. Also, press Esc to exit -O, O-Sequence menus without making a selection. However, do not use Esc to exit the -O, Make option. You must press -O and select EXIT.

@: Press the @ key to insert pauses while making an O-Sequence. When the O-Sequence is run and the "@" character is reached, operation pauses with an @ prompt until the space bar is pressed.

-K: If you press -K while making an O-Sequence, the character has no immediate effect; it is merely saved as a character in the O-Sequence. When you run the O-Sequence, -K acts as a toggle for menu suppression. After the first -K is encountered, the menus at the bottoms of displays no longer appear. Selections made while you were making the O-Sequence are still implemented, however. This allows for the implementation of commands without distracting the viewer with menu selections that flash by quickly.

A subsequent -K allows menus to appear again. Menu suppression is turned off when the ? prompt returns at the end of an O-Sequence, or when you chain to a new O-Sequence.

-J: If you press -J while making an O-Sequence, the character has no immediate effect. It is saved as a character in the O-Sequence. When you run the O-Sequence, -J suppresses all character printing on the Graphics display. Graphs are still plotted, but no values or expressions are printed anywhere on the display until a subsequent -J is encountered in the O-Sequence.

-J is only for the Graphic mode. You should always ensure that character printing is activated when on a text display—i.e. Array, Calculator, or Help Screens. Suppression of character printing is turned off when the ? prompt returns at the end of an O-Sequence.

Z: If you press the Z key at the @ prompt while making an O-Sequence, you may type a message to appear at the bottom of the display. Press Return to finish the message. This should usually be followed by the @ key to insert a pause. When the O-Sequence is run, the message is reproduced, and the pause allows time to read it.

When the character "Z" is encountered during an O-Sequence run, the sound is toggled OFF while the subsequent title is typed. All "Z-messages" are typed silently.

⌘-O: Press ⌘-O while making an O-Sequence to terminate the Make option. You must select either CHAIN, SAVE, or EXIT from the menu that appears, as described above under Make.

Filenames: Operation filenames use only the characters 0-9, A-Z, . , and - . They may be no more than 8 characters long.

WHILE Conditions: If ":W" is appended to a RUN FILE or CHAIN FILE NAME, you are prompted to enter a WHILE condition. This must be a valid BASIC logical expression, as described in Appendix 2. The O-Sequence is repeated if the expression is true (has value 1) at the end of the O-Sequence run.

Loop Index: You can cause an O-Sequence to be repeated *n* times by appending *:n*—where *n* is a positive integer constant—to a RUN FILE NAME or CHAIN FILE NAME,

Other Characters: All keys except @ and ⌘-O have the same effect while making an O-Sequence as during normal operation of *Graphics Calculator* .

When an O-Sequence is run, only space bar, Esc, and Ctrl-Reset are accepted from the keyboard. All other characters are received from the O-Sequence and have the same effect as during normal operation (except "@" and "⌘-O").

⌘-P, PROJECT/PLOT

Purpose

Plot all or parts of graphs of functions in Function format, and parametric equations in Parametric format. Project the coordinates of points as they are plotted. Assign coordinate values as current values of parameters for later calculations. Trace the mapping of function domains onto ranges. Zoom in on important points of graphs.

Operation

In Function format, press ⌘-P and use the cursor keys ← or → to select lines or arrows from the menu:

```
PROJECT/PLOT USING:    LINES    [ARROWS]
```

If you select LINES, subsequent projections are displayed with horizontal and vertical projection lines. If you select ARROWS, the projections are displayed with small arrows replacing the lines.

Upon completing your choice of lines or arrows, you will encounter the following menu:

```
FOR:   [F]   G   H   R   S   Q
```

When you have made this selection, the current value of the independent variable X, and the corresponding value Y = F(X)—assuming you selected the function F—appear at the top of the Graphics display. Each time you press the → cursor key, the value of X is in-

cremented by the current Delta-X, and Y is updated. Also, the corresponding points are plotted on the graph (if they are inside the coordinate window), and the projections are displayed as either lines or arrows. If you press ←, X is decremented by Delta-X.

To plot points more rapidly, hold down the ← or → cursor key. The last values of X and Y are displayed when the key is released. To exit the Project/Plot option, press Esc.

In Parametric format, the independent variable is T, and the parametric equations X = X(T), Y = Y(T) are graphed. Projections begin immediately upon your selection of lines or arrows.

A T-interval is displayed at the lower left of the screen, with a small arrow pointing at the current location of T. The current values of T, X, and Y appear at the lower right, upper left, and upper right, respectively. When you press ← or →, T is decremented or incremented, respectively, by the current value of the T-increment, Delta-T.

If a calculated point is outside the coordinate window in either Function or Parametric format, a small arrow points to its location, and a sound is emitted.

Second-level Input

Esc: Press Esc to exit the Project/Plot option.

X: If you press X in Function format, the current value of X is presented for changing. You may type any valid BASIC expression for X, as described in Appendix 3. When you press Return, the value of the expression is assigned as the current value of X, and that value is projected. If the value of X is not in the current X-interval, X is changed to the nearest interval endpoint, and you are notified.

T: If you press T in Parametric format, you may assign the current value of T, just as you assign X in Function format. If the value of T is not in the current T-interval, T is changed to the nearest interval endpoint, and you are notified.

Delta-X: If you press Ó-X in Function format, the current expression for the X-increment, Delta-X, is presented for changing. You may type any valid BASIC expression, as described in Appendix 3. The value of the expression is assigned to Delta-X and is used in subsequent projections.

Delta-T: In Parametric format, press Ó-T to assign the value for the T-increment, Delta-T, just as you assign Delta-X in Function format.

Ó-F: If you press Ó-F in Function format, you may use the cursor keys ← or → to select a different function—F, G, H, Q, R, or S—to graph. Points on the graph of the new function are projected.

Ó-M: If you press Ó-M, the Mapping Trace is implemented. Now, when you press ← or → to project values, the traces of X and Y are plotted on the X and Y axes, respectively. To turn the Mapping Trace off, press Ó-M again.

Ó-V: If you press Ó-V in Function format, the current values of X and Y are recorded in the table at the right of the Graphics display. In Parametric format, the values of X, Y, and T are displayed.

Ó-Z: Press Ó-Z to zoom in or out on the currently projected point. You are prompted to enter X-MAGNIF, the X-magnification factor. The next prompt asks for the Y-magnification factor. A new coordinate window is displayed, with the currently projected point at its center. Press the cursor keys ← or → to plot and project your graph again.

The width and height of the coordinate window depend upon the magnification factors used. To zoom in, use factors greater than 1. To zoom out, use factors between 0 and 1.

In Function format, Delta-X is automatically multiplied by the reciprocal of the X-magnification factor, so that it is appropriate for the new coordinate window. You may change it using ⌘-X. In Parametric format, Delta-T is not changed automatically when you zoom. You may change it using ⌘-T.

Parameters: If you press a parameter key—A, for instance—in Function format, the menu:

A = [X] Y

appears in the table at the right of the Graphics display. Use the cursor keys ← or → to select X or Y. When you press Return to complete the selection, the current value of X or Y is assigned as the current value of A (or any other parameter you have chosen), and the value is recorded in the table. The current *expression* for A is not changed by this method of assignment.

In Parametric format, you may assign the value of X, Y, or T to the parameter of your choice.

⌘-Q, QUIET OPERATION

Purpose
Turn the sound off or on.

Operation
Press ⌘-Q and use the cursor keys ← or → to make a selection from the menu:

QUIET OPERATION: NO [YES]

If you select YES, all sound is turned off. If you select NO, sound is turned on. If you press Z to type a message at the bottom of the screen (perhaps while making an O-Sequence), the sound is turned off while you type, and on when you press Return to finish the message. The "Z-message" is also typed silently when you run the O-Sequence.

⌘-R, RESTART

Purpose
Restart *Graphics Calculator* with the default setup.

Operation
Press ⌘-R and use the cursor keys ← or → to select YES or NO from the menu:

RESTART/DEFAULT SETUP: [NO] YES MENU

If you select YES, the initial Graphics display is loaded from the disk in disk drive 1, and the setup is returned to the default status. This can be either the *Graphics Calculator* program disk or an Auxiliary Disk. You cannot RESTART without either of these disks in drive 1.

If you select NO or press Esc, you are returned to the display at which ⌘-R was pressed. If you select MENU, the initial *Graphics Calculator* menu is presented. If you do not have the program disk in drive 1 when you select MENU, you will be prompted to insert it.

⌘-S, SETUP

Purpose
Display the current expressions and/or values for parameters, variables, and functions; load setup files from Auxiliary Disks; save setup files on Auxiliary Disks; display setup file catalogs; delete setup files; and print Graphics displays. All information that appears on the Calculator and Setup displays is saved. You have the option of saving the Graphics display also. You may load setups for function definitions, parameters, or commands separately if you choose.

Operation
Press ⌘-S and use the cursor keys ← or → to make a selection from the menu:

```
SETUP: [STATUS]  LOAD  SAVE  CAT  DEL  PRINT
```

Press Return to enter your selection.

STATUS: If you select STATUS, The first of four Setup/Status displays appears. Press the space bar to cycle through these displays. You can also press the → cursor key to move ahead and the ← cursor key to move back. Press Esc to exit.

The first display shows all the current parameter expressions. The second display begins a listing of command expressions and their current values. The third display continues this listing, beginning with T-INT LEFT and T-INT RIGHT. The last display shows the definitions of eight functions.

You can step quickly through the Setup displays whenever you need to see a summary of the current Setup/Status. Just press ⌘-S at the ? prompt, then press Return.

LOAD: if you select LOAD, you are prompted to enter NAME OF FILE. Type the name of a setup file that appears on the setup file catalog of the current disk. Press Return to enter the filename. Then use the cursor keys ← or → to make a selection from the menu:

```
LOAD:   [ALL]  FUNC  PAR  CMND  GRPH
```

Press Return to enter your selection. The selection GRPH appears only if Graphics was saved with the setup file.

If you select ALL, then all function expressions, parameter expressions and values, command expressions and values, and graphics are loaded. If you select FUNC, PAR, or CMND, only the corresponding expressions and values are loaded. If you select GRPH, both the Graphics display and corresponding command expressions and values are loaded. When the load is complete, you are returned to the display at which ⌘-S was pressed.

SAVE: If you select SAVE, you are prompted to enter NAME OF FILE. Type the name for your setup file, and press Return to enter the name. Use only characters listed in Appendix 1 as valid for filenames.

Now use the cursor keys to select YES or NO to answer ARE YOU SAVING Graphics? Press Return to enter.

Similarly, answer YES or NO to ARE YOU SUPPLYING A TITLE? If you select NO, your setup file is saved. If you select YES, type your title at the bottom of the display. Use up to 34 characters. When you press Return to enter the title, your setup is saved on the current Auxiliary Disk under the setup filename you supplied. Each time you load this setup, your title is printed at the bottom of the display.

When the SAVE is complete, you are returned to the display at which ⌘-S was pressed.

CAT: If you select CAT, the catalog of setup filenames is displayed for the disk in the current disk drive (depending upon the ⌘-D, DISK DRIVE setting). The right-hand column, under Graphics, indicates whether or not the corresponding setup file has a Graphics display saved with it. The original menu still appears at the bottom of the display. Now you can make selections while looking at the catalog. Press Esc to exit the catalog.

DEL: If you select DEL, you are prompted to enter NAME OF FILE. Type the setup filename you want to delete and press Return. The corresponding file is deleted from the disk in the current disk drive, and you are returned either to the ? prompt or the catalog display.

PRINT: If you select PRINT, the next prompt depends upon the printer setup you have chosen (Selection 5, Printer Setup, on the initial *Graphics Calculator* menu). If you have indicated that your printer and card accept single string commands, your default printer string is presented for possible changes. Control characters are indicated with reverse video.

If you do not want to use the default printer string, type your new printer string exactly as your system requires. (Note that the usual editing keys ⌘-D, ⌘-I, and ⌘-Q do not work here. You can use ←, →, and the space bar to change your entry.)

When you press Return, your printer string is sent to the printer card in slot 1. Be sure the printer is on line and is loaded with paper. Otherwise, the program stops, and you must press Ctrl-Reset to reboot. When printing is complete, the ? prompt returns.

If you have indicated that your printer and card do not accept single string commands, you are prompted to enter the printer program. Your default program name is presented for modification. You can use the editing keys to change it. When ready, press Return to enter the printer program name. The computer runs the program you named if it is on the disk in the current disk drive. Otherwise, a FILE NOT FOUND error is generated.

What happens next depends upon your program. For more on printer programs, see *Printing Graphics Calculator Displays* in Part II, or see Appendix 9.

Second-level Input

SETUP filenames: Only the characters A–Z, 0–9, . , and – are allowed in setup filenames. They can be at most 8 characters long.

Printer Strings: The characters in Appendix 1 for Titles, Labels, and Remarks, as well as all control characters, are allowed in printer strings. Up to seventeen characters are allowed. The ⌘-D, ⌘-I, and ⌘-Q *editing* keys are not available when typing printer strings.

Printer Program Names: The characters A–Z, 0–9, . , and – are allowed. The name can be up to 8 characters long.

Esc: The Esc key exits any menu or entry option and returns you to the ? prompt. The Esc key has no effect while printing is in progress. To halt printing while in progress, use the Ctrl-Reset key to reboot the system.

♂-T, T-INCREMENT

Purpose
Change the value of the T-increment, Delta-T, used for Project/Plot and Arrays in Parametric format.

Operation
Press ♂-T and type a valid BASIC algebraic expression (or change the current expression) when the Delta-T prompt appears. Press Return to complete your entry. The value of the expression you enter must be positive and less than 1.8E +38. If not, the entry is rejected, and the Delta-T prompt reappears. Of course, the value should also be reasonable in relation to the current coordinate window, or the graphic effect may not be desirable.

You may press Esc to exit the ♂-T, T-increment option without changing Delta-T.

♂-U, UNCONNECT OR CONNECT POINTS

Purpose
Select unconnected or connected points for plotting graphs.

Operation
Press ♂-U and use the cursor keys ← or → to make a selection from the menu:

```
UNCONNECT/CONNECT POINTS:   CON   [UNCON]
```

Press Return to complete your selection.

If you select UNCON, only calculated points are plotted on subsequent graphs. If you select CON, the calculated points are plotted, and each time two consecutive successful calculations are completed, the corresponding points are connected by a line segment. (The segment is clipped to fit the coordinate window, if necessary.)

♂-V, VALUES ASSIGNED

Purpose
Plot individually selected points on the graph of a function or parametric equations and record the corresponding coordinates in the table on the right side of the Graphics display. Also, plot polygons or polygonal line segments with selected vertices.

Operation
In Function format, press ♂-V and use the cursor keys ← or → to make a selection from the menu:

```
VALUES  ASSIGNED:   [F]   G   H   R   S   Q   POLY
```

Press Return to complete your selection.

If you select F, G, H, R, S, or Q, a cursor flashes at the upper left of the Graphics display, prompting you to enter a value for the independent variable, X. You may enter any valid BASIC expression, up to 14 characters, as described in Appendix 2. The value of the expression is assigned as the current value of X. The function value—F(X), for instance—is assigned as the current value of the dependent variable, Y. The values of X and Y are also recorded in the table at the right side of the display. If more than 6 characters are needed, the sixth character is a question mark (?), and only the first 5 characters are shown. At the same time, the point corresponding to the pair (X,Y) is plotted on the

coordinate window (if it is inside the current coordinate window). The points are plotted double-size and always in the color white. Then you are prompted to enter another value for X. The points are not connected.

When you have assigned all the values you want, press Esc to exit the option.

If you select POLY, then enter expressions for both X and Y. Successive points are connected or not, depending upon whether the ♂-U option is set on CON or UNCON, respectively. The color of the connecting segments is determined by the ♂-C, Color of Graph setting.

In Parametric format, the menu is:

VALUES ASSIGNED FOR X(T), Y(T): NO [YES]

If you select YES, then enter an expression for T at the lower right of the Graphics display. The corresponding values for X and Y are shown at the top of the display. The values of X, Y, and T are recorded in the right-hand table (truncated, if necessary, as described above), and the point (X,Y) is plotted double-size in white. Successive points are not connected.

Press Esc when you have entered all the T-values you want.

♂-W, WINDOW COORDINATES

Purpose

Change the scale and location of the coordinate window on the Graphics display. Change the distance between tick marks on coordinate axes.

Operation

Press ♂-W and use the cursor keys ← or → to make a selection from the menu:

WINDOW COORDS: [BOUNDS] FRAME

Press Return to complete your selection.

With the BOUNDS selection, you enter the left, right, bottom, and top bounds of the coordinate window. With the FRAME selection, you frame a section of the current coordinate window which then expands to fill the coordinate window.

If you select BOUNDS, you are prompted to enter a value for the left side. Type a valid BASIC algebraic expression. When you press Return to complete this entry, the value of the expression is assigned as the left side of the coordinate window, and the window is labeled with this value. Then you are prompted to enter the right side value The expression you enter must have a value greater than that of the left side. If not, the entry is rejected, and the prompt reappears.

After the left and right sides, enter the bottom and the top. The value of the expression you enter for the top must be greater than the value for the bottom. Otherwise, the entry is rejected, and the prompt to enter the top reappears. To start over at any point, press Esc and ♂-W again.

After you enter the expression for the top, the coordinate window is erased, and new axes are drawn if they are inside the new coordinate window. Then you are prompted to enter X-tick. If you are satisfied with the tick marks on the axes just drawn, press Esc to exit the ♂-W, Window Coordinates option. If not, type a valid BASIC algebraic expression for the distance between tick marks on the X-axis. When you press Return to complete your entry, you are prompted to enter the Y-tick. Type an expression for the distance between tick marks on the Y-axis. When you press Return, new axes are drawn with the tick marks you selected. If you ask for tick marks too close together, they are not drawn. (No more than 30 tick marks are allowed on each axis.)

If you select FRAME from the menu:

```
WINDOW  COORDS:    BOUNDS    [FRAME]
```

and press Return to complete your selection, the prompt "USE ARROWS TO MOVE LEFT SIDE" appears. Each time you press →, the left side border of the coordinate window moves to the right as the left side label is incremented by the current value of Delta-X. To move the border quickly, hold down the → key. The ← key moves the border left but you cannot move it left of its original position. When the left border is in the position you want, press Return to fix this position.

Then, the prompt "USE ARROWS TO MOVE RIGHT SIDE" appears. Press ← to move the right border to the left and → to move it to the right. You cannot move it past the left border or to the right of its original position. Press Return to fix the position when you are ready.

Next, you are prompted to "USE ARROWS TO MOVE BOTTOM." Press ↑ to move it up and ↓ to move it down. Press Return to fix the position. Then you are prompted to "USE ARROWS TO MOVE TOP." Press ↓ to move it down and ↑ to move it up. Again, press Return to fix the position.

The coordinate window is erased, and the axes are redrawn if they are inside the new coordinate window. Then you are prompted to enter the X-tick. Tick marks are assigned just as described above for the Bounds selection. Press Esc to leave the tick marks as they are, or enter the X-tick and Y-tick distances as described.

Second-level Input

Esc: Press Esc if you wish to end the ♂-W, Window Coordinates routine before completing all entries. If you press Esc before entering the top (or fixing the top border), the old coordinate window is restored after it is erased. If you press Esc after pressing Return to complete the top entry, you will get the new coordinate window with the old tick mark distances.

X: If you press X in the Frame option while in the process of locating the left or right border, you can assign the border location exactly by typing a number or other valid BASIC algebraic expression. When you press Return to complete the entry, the border moves to the location you specified, if possible. If not, X is changed automatically, and you are notified.

♂-X: If you press ♂-X in the Border option while in the process of locating the left or right border, you can change the value of the X-increment, Delta-X, used to move the border. Type a positive number or valid BASIC algebraic expression with a positive value. After you press Return to complete the entry, the expression becomes the current expression for Delta-X, and its value is the current value of Delta-X used to move the border.

Y: If you press Y in the Frame option while in the process of locating the top or bottom border, you can assign the border location exactly by typing a number or other valid BASIC algebraic expression. When you press Return to complete the entry, the border moves to the location you specified, if possible. If not, Y is changed automatically, and you are notified.

♂-Y: If you press ♂-Y in the Frame option while in the process of locating the bottom or top borders, you can change the value of the Y-increment, Delta-Y, used to move the border. Type a positive number or other valid BASIC algebraic expression with positive value. After you press Return to complete the entry, the value of the expression becomes the current value of Delta-Y, and the expression is the current expression for Delta-Y.

⌘-X, X-INCREMENT

Purpose
Assign the current expression and value for the X-increment, Delta-X.

Operation
You can assign Delta-X at the ? prompt, or you can assign Delta-X after starting an option that uses it.

If you press ⌘-X, the prompt Delta-X appears. Type a positive number or other valid BASIC algebraic expression whose value is positive. After you press Return to complete the entry, the expression is assigned as the current expression for Delta-X, and its value is the current value. This value is used in all options involving Delta-X until you change it.

If the expression you enter does not have a positive value, it is rejected, and the Delta-X prompt is repeated. To exit the ⌘-X, X-increment option before pressing Return, press Esc. Then Delta-X is not updated.

⌘-Y, Y-INCREMENT

Purpose
Assign the current expression and value for the Y-increment, Delta-Y.

Operation
You can assign Delta-Y at the ? prompt, or you can assign Delta-Y after starting an option that uses it.

The operation of the ⌘-Y, Y-increment option is the same as for the ⌘-X, X-increment option, except that you press ⌘-Y to begin.

Appendix 7: Error Messages

Graphics Calculator is designed with complete syntax checking and error control. After the system has been successfully booted with the *Graphics Calculator* disk in disk drive 1, the program should trap all errors not related to hardware malfunction and allow you to recover.

There are three kinds of error indicators: ignoring characters, repeating input prompts, and error messages.

The first indicator prevents errors by allowing you to use only appropriate characters. The second indicator means you have entered an inappropriate value and it has been rejected. The third indicator is a message that gives a brief description of what is wrong.

Ignoring Characters

If there is no response when you press a key or control character, this means that the corresponding character cannot be used in the current situation. In many options, the *Graphics Calculator* simply ignores keys that are inappropriate.

Repeating Input Prompts

If you press an inappropriate key at the ? prompt, the message INVALID CHOICE appears, and the ? prompt returns. If the previous input prompt is repeated when you press Return to complete an entry, this means your entry has been rejected. This happens, for example, if you enter a negative value for the X-increment, Delta-X. Since Delta-X must be positive, your entry is rejected. The input prompt reappears so that you can enter a new value. In some options, the input prompt is repeated if you enter an undefined value, or if an overflow error occurs. If you cannot determine an appropriate value to re-enter, press Esc to return to the ? prompt. Then you may select the option again and start over.

Error Messages

For many errors, a message is presented on the screen that describes the error. Below we list each error message and briefly discuss probable causes.

INVALID KEY

This message appears when you press an invalid key at the ? prompt. The keys valid at the ? prompt are discussed in Part I of this manual, in the chapter titled *Interactive Use—A Quick Start.* The flashing ? prompt returns immediately with this error message, so that you can continue normal operation.

YOUR ENTRY IS INVALID:
2X
PRESS SPACE BAR TO REENTER.

This message indicates a syntax error. Your entry does not conform to correct BASIC syntax. In the above, for example, there must be an * symbol between 2 and X to indicate multiplication. The invalid entry is always presented in reverse video with the error message. When you press the space bar, the prompt to enter reappears, and the invalid entry is presented for correction. For more details on correct syntax, see Appendices 2, 3 and 8.

USE ONE LETTER VARIABLES & PARAMETERS!
(A-E, I-P, T-Y)
A * XX
USE * WHERE NEEDED
PRESS SPACE BAR TO REENTER

This message occurs when you use a variable or parameter such as XX. Only single-letter variables and parameters are allowed on *Graphics Calculator*. Omitting an asterisk (*) symbol for multiplication is a frequent cause of this error. Note that F, G, H, R, S, and Q are not variables or parameters; they are reserved for function names. Also, Z may not be used in algebraic expressions.

When you press the space bar, the invalid entry is presented for correction.

YOUR ENTRY IS INVALID:
CHECK ALL FUNCTIONS FOR SYNTAX ERRORS
PRESS SPACE BAR TO CONTINUE

This message appears on the rare occasion in which an invalid function is not detected at the time of entry. Later, when the function is used, the error is discovered, and you are notified. Since several functions may be used at once (in composite function definitions, for example), you may need to check all six functions.

When you press the space bar to continue, the ? prompt returns so that you can check all functions. This is done most easily on the Calculator display (press ₲-M).

UNDEFINED VALUE IN ENTRY 1 / X CURRENT VALUE NOT UPDATED PRESS SPACE BAR TO CONTINUE

When this message occurs, the syntax in an algebraic expression is correct, but the expression cannot be evaluated. For example, if X = 0, the expression 1/X cannot be evaluated. Or, if A < 0, SQR(A) is undefined. If you intended to assign the value of the expression as the current *value* of a variable or parameter, the assignment is not made. The expression does, however, become the current *expression* for the parameter and may be used later when the value of X or A, for example, is changed. When you press the space bar to continue, the ? prompt returns.

OVERFLOW ERROR IN ENTRY 1 E + 39 CURRENT VALUE NOT UPDATED PRESS SPACE BAR TO CONTINUE

When this message occurs, the syntax in an algebraic expression is correct, but an overflow error has occurred during its evaluation. Just as with undefined values, the current value is not updated but the current expression is. The ? prompt returns when you press the space bar .

YOU CAN'T ASSIGN UNDEFINED OR OVERFLOW VALUES! PRESS SPACE BAR TO CONTINUE

This message appears when you try to assign a value that has already been specified as an undefined or overflow value.

For example, if Y = UNDEFINED in the ♂-P, Project/Plot option, you should not press A and select A = Y. When you press the space bar, the option continues.

> OUT OF MEMORY!
>
> CHECK FOR A FUNCTION SELF-REFERENCE
> OR A FUNCTION THAT IS TOO COMPLEX
>
> PRESS SPACE BAR TO CONTINUE

This message appears when the stack is full. Function self-reference is the most frequent cause. For example, if you try to define F(X) = 1 + F(X), you will get this message. Indirect self-reference such as F(X) = G(X), G(X) = H(X), H(X) = F(X) also causes this error.

Infrequently, a very complicated function produces the OUT OF MEMORY error. For example, define:

F(X) = A + X * (B + X * (C + X * (D + X * (E + N * X)))).

Now, if you try to define M = F(2), the OUT OF MEMORY error appears. You can generally use combinations and compositions of functions to avoid such complications.

The ? prompt returns when you press the space bar to continue. You should change the function definitions that caused the error before you continue.

> X (or T) HAS BEEN CHANGED TO START (OR END) OF INTERVAL
>
> PRESS SPACE BAR TO CONTINUE

This message appears in the ⌘-P, Project/Plot option if you try to assign the current value of the independent variable, X or T, outside the current interval. (See ⌘-I, Interval Choice.) To project, the value of X or T must be in the current interval. If it is not, the nearest endpoint of the current interval is assigned automatically as the current value of X or T, and you are notified by the above message . The Project/Plot option continues when you press the space bar .

> X (or Y) HAS BEEN CHANGED TO FIT IN WINDOW
>
> PRESS SPACE BAR TO CONTINUE

In the ⌘-W, Window Coordinates option, if you select the Frame option, you can assign values to the variables X and Y to locate the sides of the frame. But these sides cannot be outside the old coordinate window, the right side cannot be left of the left side, and the top cannot be below the bottom. If you assign a value to X or Y that violates one of these conditions, the above message appears, and X or Y is adjusted to the nearest correct value. When you press the space bar , the Frame option continues.

YOUR DISK IS WRITE PROTECTED! PRESS SPACE BAR TO CONTINUE

This message appears if you try to save a setup file or an O-Sequence file on a disk that is write-protected. The notch near the disk label is covered. This protection must be removed before you can save a file. When you press the space bar , the ? prompt returns.

NO GRAPHICS CALCULATOR OR AUXILIARY DISK IN DRIVE #1 (OR 2) PRESS SPACE BAR TO CONTINUE

This message appears if there is no *Graphics Calculator* or Auxiliary Disk in the current disk drive when the program tries to access that drive. Press ⌘-D to select 1 or 2 as the current drive. If you have only one drive, never select 2. Also, be sure that the drive door is closed and a disk is in the current drive before you press the space bar to continue. When you press the space bar, the ? prompt returns.

FILE NOT FOUND PRESS SPACE BAR TO CONTINUE

This message appears when the setup file or O-Sequence file you ask for is not found on the *Graphics Calculator* or Auxiliary Disk in the current disk drive. (See ⌘-S, Setup or ⌘-O, Operation Sequence.) When you press the space bar , the ? prompt returns. Then you can insert a disk with the needed file in the drive and repeat the ⌘-S or ⌘-O command.

YOUR DISK IS FULL! PRESS SPACE BAR TO CONTINUE

If there is not enough space left on a disk to save a setup file or an O-Sequence file, the above message appears when you try to save it. When you press the space bar to continue, the ? or @ prompt returns. Then you can insert another Auxiliary Disk and repeat the command to save the file (press ⌘-S or ⌘-O , respectively).

FILE NAMES MUST BEGIN WITH A LETTER AND MAY ONLY
CONTAIN THE SYMBOLS:

A-Z, 0-9, . , -

PRESS SPACE BAR TO CONTINUE

This message appears when you use an invalid filename while saving a setup file or an O-Sequence file. (See ♂-S, Setup or ♂-O, Operation Sequence.) Note that only letters of the alphabet A-Z, digits 0-9, period, and minus sign may be used. When you press the space bar to continue, the prompt to enter NAME OF FILE returns so that you can enter a valid filename.

UNKNOWN ERROR - YOU MAY NEED TO REBOOT

PRESS SPACE BAR TO CONTINUE

This error message appears when no other error message applies. It should appear only if there is a hardware or software malfunction. For example, static electricity could cause this message. We hope the software never malfunctions, but there is always the possibility of an undiscovered "bug."

When you press the space bar to continue, the program may recover if the error was not too severe. Otherwise, press Ctrl-Reset to reboot the system. Be sure you have a *Graphics Calculator* program disk (not an Auxiliary Disk) in drive 1 before rebooting.

Appendix 8: Beginning BASIC

Graphics Calculator uses the notation (syntax) of the BASIC computer language for the expressions that define functions and assign values to parameters and variables. In many cases, the symbols and their uses are the same as you learned in algebra. There are some differences, however.

Multiplication and Exponents

Two important differences between algebra and BASIC notation are:

* is used for multiplication and cannot be omitted.
^ is used for exponents.

Here is a list of examples showing the use of these two symbols:

ALGEBRAIC NOTATION	BASIC NOTATION
$3X - 2$	3 * X - 2
Y^2	Y^2
$3(2X - 1)^3$	3 * (2 * X - 1)^3
$-T^2$	-(T^2)
$A^2 + B^2$	A^2 + B^2

Notice that whenever multiplication is indicated by juxtaposition in algebraic notation, an * has been used in BASIC notation. Also, each exponent in algebraic notation has been preceded by a ^ in BASIC notation.

Order of Precedence

Calculations are carried out in a prescribed order when an expression is evaluated. This order is called the *order of precedence.* In general, this is the same order as you use in algebra. All exponentiations are done first, proceeding from left to right in the BASIC expression. Next, the multiplications and divisions are done from left to right. Finally, additions and subtractions are done from left to right in the expression.

For example, suppose the value of A^2 + B^2 + C^2 is calculated. First, A^2, B^2, and C^2 are calculated. Then, A^2 and B^2 are added, and C^2 is added to the result.

The order of precedence can be controlled by the use of parentheses, just as in algebra. When the expression A + (B + C) is evaluated, B and C are added first, then the result is added to A.

Division

In BASIC the symbol / indicates division. There is no horizontal bar for fractions as in algebra. BASIC fractions are expressed horizontally on a single line. For this reason, you must be careful about using parentheses to group numerators and denominators separately. Here are some examples:

ALGEBRAIC NOTATION	BASIC NOTATION
$\frac{A+B}{C}$	(A + B)/C
$\frac{A}{B-C}$	A/(B – C)
$\frac{A}{B}-C$	A/B – C
$\frac{\frac{A-B}{C}+D}{U+V}$	((A – B)/C + D)/(U + V)

Limiting expressions to one horizontal line in BASIC makes fractions and exponents a little more difficult to deal with. When in doubt, control the order of precedence with parentheses.

Numbers

Numbers are represented in BASIC output with floating point or exponential form. Some examples of floating point form are –12.345, .01234, and –.123456789.

Exponential examples are 1.2E –5, 1.23456E +7, and –1.23456789E –36. The numbers following the E in these expressions are exponents and are the same as exponents used in scientific notation. This notation is equivalent to scientific notation. For example, .000012 is displayed as 1.2E –5, and 123456789000 as 1.234565789E +11 in BASIC. The same numbers in scientific notation are written as $1.2\text{x}10^{-5}$ and $1.23456789\text{x}10^{+11}$.

If possible, the exponent is avoided by moving the decimal point in the mantissa. For example, the floating point form 1234.56789 is displayed rather than the exponential form 1.23456789E +03, and .01234 is used rather than 1.234E –02. However, no more than one leading zero is displayed in the mantissa when floating point form is used, and trailing zeros in the mantissa are never printed. For example, .001234000 is printed as 1.234E –03.

Built-in Functions

BASIC does not use the radical symbol ($\sqrt{\ }$) to indicate square roots. Rather, $\sqrt{X}$ is denoted by the square root function, SQR(X). Similarly, absolute value, | X |, is denoted by ABS(X).

SQR and ABS are examples of BASIC built-in (or intrinsic) functions. The trigonometric functions sine, cosine, and tangent are also available as BASIC functions. A complete list of BASIC built-in functions appears in Appendix 4, as well as on the *Graphics Calculator* Help screens.

Practice

You can define almost any expression you need using arithmetic operators and BASIC built-in functions. The best way to learn is to practice. We suggest that you work the examples in the introductory sections of this manual first. Then, get out your favorite algebra or precalculus book, and try using some of the functions or expressions you find there on *Graphics Calculator*. Be sure to translate them into BASIC notation. Experimentation is easy because *Graphics Calculator* checks syntax and lets you correct it. Try reproducing some of the function graphs in your text on *Graphics Calculator*.

You might want to consult Appendices 3, 4, and 5 of this manual as you practice. They provide complete details for using BASIC expressions to define functions and parameters on *Graphics Calculator*.

Appendix 9: Printing with *Graphics Calculator*

Printing Graphics Displays

If your printer accepts single-string commands (an Epson™ FX-80 with a Grappler™ card, for example), you can print Graphics displays while the *Graphics Calculator* program is running. If your printer does not accept single-string commands (the Apple Imagewriter, for example) you can save setups with Graphics displays on Auxiliary Disks and print the graphics screens later. A third alternative is to use a screen dump printer interface card that prints any screen without interrupting the program. (See the section *Screen Dump Printer Interface Cards* at the end of this appendix.)

Printer Setup

You can configure *Graphics Calculator* to use single-string printer commands, which can be executed from within the program, or you can configure it to run your own printer-driver software. To do either, remove the write-protection tab from the *Graphics Calculator* disk, insert the disk into drive 1, and turn the computer on. At the first menu, press 5, Return to select Printer Setup. The following setup menu appears:

```
Present print string is:

   [IMAGEWRITER]

This is default for an Imagewriter
with a Super Serial Card or standard
Apple IIc and IIgs connections.

Printer is in slot number: [1]

  1. Quit to the menu.

  2. Enter new single-string command.

  3. Enter new program name.

  4. Enter new slot number.

     Your choice: [1]
```

Single-String Printer Commands

If you press 2, Return at the above menu, the following prompt appears at the bottom of the screen:

PRINTER STRING:

You must enter the string of command characters that you want to send to your printer card (in slot 1) to print Graphics displays. For example, if you have an Epson MX-80 or FX-80 printer with a Grappler card, you could enter "[CTRL-I], G,R,I,D" as your default command, which will appear as [I]GRID.

Note that control characters appear in reverse video on the screen. Also note that spaces are important. If your printer does not accept spaces in its command string, don't enter any spaces.

When you press Return to complete your printer command entry, the string will be saved on your *Graphics Calculator* main disk for later use. (Don't forget to put the write-protection tab back over the notch on the *Graphics Calculator* disk.) Press 1, Return to go back to the initial *Graphics Calculator* menu.

Printer Driver Programs

Some printers and cards require a printer driver program to print graphics; you must interrupt the *Graphics Calculator* program to use such printers (which include the Apple Imagewriter® with the Super Serial Card.) With these printers, you must save setups with Graphics displays on Auxiliary Disks before you print them. You can save several screens and then interrupt the *Graphics Calculator* program to print. You can run your printer driver program directly from the *Graphics Calculator* program.

If you are going to use a separate printer driver program, choose option 3 at the setup menu. When the menu prompt

PRINTER PROGRAM:

appears at the bottom of the screen, type the filename of your program. For example, there is a program on the Auxiliary Disk called Imagewriter. You can use this program with your Imagewriter printer. When you press Return, the filename is saved on the *Graphics Calculator* disk for later use. Then choose option 1 to go back to the initial *Graphics Calculator* menu. Install the write-protection tab on the *Graphics Calculator* disk before proceeding.

If you have a different brand of serial printer, it may be possible to modify the Imagewriter program to interface the *Graphics Calculator* ♂-S, Setup menu system with your own printer driver program. There is a copy of the Imagewriter program on your *Graphics Calculator* Auxiliary disk. In that program, line 100 appears as:

```
100 PRINT FI$ ="IMAGEII.6C00"
```

If you change IMAGEII.6C00 to the name of your own binary printer driver program, it will be loaded instead.

Line 108 appears as:

```
108 CALL 27648
```

This line calls the binary program named in line 100. You should change 27648 to the address used by your binary program. Then save the modified Imagewriter program on an Auxiliary Disk with the printer program filename you want to use above when you set up *Graphics Calculator* for printing. Also, save your binary program. Of course, the disk you save it on must be in drive 1 when you want to use it.

Printing with Single String Commands

Load *Graphics Calculator* in the usual way. Be sure your printer is ON and has paper in it. Then, at a Graphics display you want to print,

press ♂-S for the menu

SETUP: [STATUS] LOAD SAVE CAT DEL PRNT

select PRNT,
press Return to complete the entry,
press Return again to enter the default printer string that appears in the menu

PRINTER STRING: [I]GRID

select NO from the menu

ARE YOU SUPPLYING A TITLE? [NO] YES

and press Return to complete the entry.

Now the printer starts printing the current Graphics display. When printing is complete, the ? prompt returns.

The default printer string presented is the one you saved earlier on the *Graphics Calculator* disk with the printer-setup procedure. You have two options when printing Graphics displays. Before pressing Return to enter your default printer command string, you can change it to any command you want. Just type over the current string, and erase trailing characters with the space bar. Remember that the control characters—♂-D, ♂-I, and ♂-Q—that you usually use with the input editor don't work here because you may need these characters in your printer string.

You also have the option of supplying a title at the bottom of the screen before you print it. To do this, select YES at the appropriate menu, and type the title you want. When you press Return to enter the title, it is automatically centered before printing begins.

Printing from *Graphics Calculator* with a Printer Program

If you configured *Graphics Calculator* for a separate printer driver program, you can run your printer driver program (or any other program) directly from the *Graphics Calculator* program.

With the *Graphics Calculator* program running, insert an Auxiliary Disk containing your printer driver program into drive 1.

At the ? prompt, and press ♂-S for the following menu:

SETUP: [STATUS] LOAD SAVE CAT DEL PRNT

select PRNT, then press Return for the menu:

PRINTER PROGRAM: IMAGEWRITER

Enter the filename of your driver program (if different from the default filename presented).

When you press Return, your program runs. What happens next depends upon your program. Your program (or you) must load graphics screens from the *Graphics Calculator* program or Auxiliary Disk on which you saved them. Then it must print these graphics screens.

If you are going to use your own printer program, you need to know that Graphics displays are saved on *Graphics Calculator* Auxiliary disks with the filename suffix ".PIC". For example, if you save a setup (including the Graphics display) with the filename "TEST," the Graphics display is actually saved in a binary file with the filename "TEST.PIC"; you or your program may need to use the name "TEST.PIC" to load the file before you print it.

We have incorporated the *Graphics Calculator* catalog system for files into the Imagewriter program. If you enter Imagewriter for your printer program name and insert an Auxiliary Disk containing previously saved Graphics screens, the program runs, and a catalog of all screen names saved on the current disk is presented. To see the catalog on a Auxiliary Disk, insert that disk and press Return.

To load any screen in the catalog, type the screen name and press Return. After the screen is loaded into the computer, press the space bar to print it, or press Esc to go back to the catalog without printing. Note that the screen names are the same names you used to save the original setups. The suffix ".PIC" is not used with this catalog system.

When you have printed all the screens you want, press Esc to exit, insert the *Graphics Calculator* program disk into drive 1, and press the space bar to reboot the program.

You may save Graphics displays on your Auxiliary Disk for printing at a later time. In that case, you may start the computer from the Auxiliary Disk and press Space Bar to choose the Imagewriter program. Or, press Control-Reset to stop, then execute your printer program by typing

RUN <YOUR PROGRAM NAME>

Screen Dump Printer Interface Cards

Printer interface cards that allow you to print a screen (graphics or text) at *any* time and with *any* program are extremely convenient. With these cards, printing is software independent. FingerPrint™ Plus, available from CONDUIT, is such a card. With a FingerPrint Plus card installed in your computer, you need only press a button whenever you want to print the current screen. If a program is running, it stops, and you are provided an elaborate menu of print options from which to choose. When printing is complete, press ESC, and your program continues exactly where it stopped.

Appendix 10: Computer Arithmetic Errors

Values computed on calculators and computers are very often not exact. Often they are good approximations to true values; but sometimes they are very poor approximations. At other times, an approximation to the true value cannot even be computed.

It is important that users of computers and calculators become familiar with the limitations of the fixed precision arithmetic that is used. Observing tables of function values on the Array display is a convenient way to get some experience.

To begin this section, press ♂-R to restart with the default setup in Function format. If you are not in Function format, press ♂-F to change. Then press ♂-M to change to the Array display.

Example 73: Many precalculus textbooks point out that the quantity $(1 + 1/X)^X$ approaches the natural logarithm base, e = 2.718281828459045 . . ., as X increases without bound. But if we try to verify this by looking at tables of values for large X, it doesn't quite happen on the computer.

First, define

F(X) = (1 + 1/X)^X

and

G(X) = F(X) – EXP(1).

Next, press

♂-X and change Delta-X to 100 and

♂-A to select F for a table of values.

Now, press → or ↓ to scroll the table, and observe the values of F(X) as X is incremented by 100. They increase quickly to approximately 2.7176 (at X = 2000) and then move more slowly. At X = 5000, the first four digits, 2.718, are correct. For larger X, near 20 000, the best approximation is 2.7182. (Just press X and enter 20 000.)

For very large X, the approximation gets worse rather than better. For example, try X = 500000. The value is 2.71901541. Then try 1000000. The value is 2.71 945 426. These are only accurate to 3 significant digits (compared to 4 significant digits at X = 5000). If you try X = 1E +20, all accuracy is lost. Note also that

Delta-X = 100 has no effect when X is this large.

Another way to investigate the error is with an array of values for G(X). Press

Esc to leave the scrolling option,

X to enter X = 0, and

♂-A to select G.

Scroll the table for G to observe the difference between F(X) = (1 + 1/X)^X and EXP(1) = e. Try the same values for X used above.

Press Esc to return to the **?** prompt.

All digital devices are subject to the limitations of fixed precision arithmetic. *Graphics Calculator* is no exception. It has been designed to use BASIC to its full accuracy, but it is not *more* accurate.

There are many sources of error in computer calculations. Usually, small-approximation errors are not a serious problem, and beginners' faith in calculated values is justified. However, you need to be aware that sources of error exist, and that they occasionally produce seriously inaccurate results. *Overflow, underflow, negligible addition, loss of significance,* and *error magnification* are the names of some of these sources. Following is a discussion of these types of errors.

Overflow and Underflow Errors

The magnitude of numbers that a computer can compute and store is limited. If values are too large, overflow occurs. If values are too close to zero, they are rounded to zero, and underflow occurs.

Example 74: We investigate overflow first, then underflow. To begin, press

F to define F(X) = 10^X,

X to define X = 38,

Ó-X and set Delta-X = 1, and

Ó-A and select F for a table of values.

Notice that for X > 38, the "overflow" message is printed in the F(X) column of the array. The largest integer value of X for which the computer can compute and store 10^X is X = 38. If you try Delta-X = .1, you will find that X = 38.2 produces a value, but that X = 38.3 produces "overflow." Now, press

Ó-X to enter 1 again, and

X to enter -38.

The exact value of the real number 10^X is always positive. But if X is an integer and X < -38, the computer says 10^X = 0. This is called "underflow." Numbers very close to zero are rounded to zero, and you are not notified. This can result in undefined values (division by zero) when you don't expect it.

By choosing appropriate X and Delta-X, you might want to try to find the largest value of X (between 38 and 39) for which a value of 10^X is computed. Similarly, try to find the least value (between -38 and -39) for which 10^X is computed as a positive value. When you are finished, press Esc to return to the **?** prompt.

Inherent Error

Inherent error occurs because computers cannot always compute or store the exact values we want. Often, only approximations to the numbers we want to use are available. The problem is compounded when these approximations are used in place of exact values for later computations.

We communicate with most digital devices using decimal (base 10) number representation. However, these machines store and compute using a different form called *binary representation.* The binary representation may not be apparent to you, because the device converts your decimal input to binary before using it, and results are converted back to base 10 before you see them.

This conversion between decimal and binary representation often introduces errors. Further errors result from rounding while calculating with, and storing, binary numbers. These errors are unavoidable, so they are called *inherent errors.* A complete explanation in terms of binary (base 2) representation of numbers is beyond the scope of this manual. However, the following examples demonstrate inherent errors.

Example 75: This example illustrates the kind of errors just described. Proceed as follows:

Press ⌘-R to restart.

Press ⌘-M to select the Calculator display.

Define	to get
A = 1/3	A = 0.333 333 333
B = 10 * A	B = 3.33 333 334
C = 10/3	C = 3.33 333 333
D = 2 * B	D = 6.66 666 667
E = 2 * C	E = 6.66 666 667
I = 3.333 333 34	I = 3.33 333 334
J = 2 * I	J = 6.66 666 668

Check the current values of A, B, C, D and E as listed above or on the Calculator display. First, notice that the displayed value of A is not exactly 1/3. The displayed value of A is a 9-digit decimal approximation to 1/3.

The displayed value of A and the value actually stored are also slightly different. The stored value is used to calculate B. If the displayed value of A were used to calculate B, you would get the value 10 * (.333333333) = 3.33 333 333 0. Since only 9 digits are displayed, you should expect B = 3.33 333 333. What you actually get for the displayed value of B is the approximation B = 3.33 333 334, correct only to 8 digits.

Check the value for C = 10/3. You should expect B and C to have the same value. The displayed value is C = 3.33 333 33—a correct 9-digit approximation, but different from the value displayed for B.

Next, compare the displayed values for B and I. They are identical. If you multiply each by 2, you should expect the value 6.66 666 668 for both D and J. But multiplying B and I each by 2 produces two different results for D and J!

On the other hand, B and C have different displayed values. Multiplying each of these by 2, you should expect different values: D = 6.66 666 668 and E = 6.66 666 666. What you get is the same displayed value, 6.66 666 667, which is different from either expected value.

Of course, in these examples, we are assuming your expectations are based only upon displayed current values.

The above example shows that the exact value of an expression, say 1/3, is often different from its displayed current value, which can in turn be different from the value stored internally for use in later calculations. In Appendix 5, we noted such a difference for the value of π used in *Graphics Calculator.*

Representation and Rounding of Displayed Values

Numbers are represented on *Graphics Calculator* displays in floating point or exponential form using the BASIC conventions. A 9- or 10-digit mantissa is used, and, where necessary, an exponent. Examples of floating point numbers are 123, 12.345, .01234, and .123456789. Exponential examples are 1.2E –5, 1.23456E –7, and 1.23456789E –36.

When exponential form is used, displayed numbers have the form $\pm n_1 . n_2 n_3 \ldots n_9$ E ± N, where $n_1, n_2, n_3 \ldots n_9$ are digits 0, 1, 2, . . ., 9, with $n_1 \neq 0$. Then $n_1 . n_2 n_3 \ldots n_9$ is called the *mantissa* of the number. The mantissa is a 9-digit decimal between 1 and 10. The leading " + " and trailing zeroes are not printed.

The exponent, N, is a positive integer less than 39. The notation E ± N means that the mantissa is multiplied by $10^{\pm N}$. This is equivalent to moving the decimal point N places to the right or the left, depending upon whether + N or –N is used. This notation is equivalent to the scientific notation used in elementary algebra. For example, .000012 is displayed as 1.2E –5 and 123456789000 as 1.234565789E +11.

If possible, the exponent is avoided by moving the decimal point in the mantissa. For example, the floating point form 1234.56789 is displayed rather than the exponential form 1.23456789E +03, and .01234 is used rather than 1.234E –02. However, no more than one leading zero is displayed in the mantissa when floating point form is used, and trailing zeros in the mantissa are never printed. For example, .001234000 is printed as 1.234E –03.

The mantissa is often obtained by rounding to 9 significant digits. For example, if the tenth digit is 4 or less, the ninth digit is not changed; but if the tenth digit is 5 or more, the ninth digit is increased by 1. This is the familiar "rule of five." But this rule is not followed consistently, as the following example shows:

Example 76: On the Calculator display, enter the given 10-digit expressions in the first column below for A, and observe the 9-digit current value displayed. The (*) cases show that rounding up can first occur when the tenth digit is 4, 5, or 6. Obviously, rounding is not consistent with the "rule of five."

Entry is easy if you assign each expression to A. You need only change the last two digits after you make the first entry. In the table, we have inserted spaces for readability.

Expressions Entered (10 digits)	Current Value Displayed (9 digits)
3.33 333 3333	3.33 333 333
(*) 3.33 333 3334	3.33 333 334
3.33 333 3324	3.33 333 332
(*) 3.33 333 3325	3.33 333 333
3.33 333 3315	3.33 333 331
(*) 3.33 333 3316	3.33 333 332

Loss of Significance and Error Magnifications

If nearly equal values are subtracted, the digits in the resulting mantissa may be quite unreliable. This is called *loss of significance*. Here is an example:

> *Example 77:* Define A = 3.33 333 3324 and B = 3.33 333 3325. Each has ten digits. The resulting 9-digit values displayed are A = 3.33 333 332 and B = 3.33 333 333. Now define C = B – A. The actual difference of the entered expressions is 1E –9. The actual difference of the displayed values is 1E –8. But the displayed value of the calculated difference, C = 2.79396772E –09, is neither of these.

Error magnification occurs when a value that is already wrong is multiplied by a large value (or divided by a small value). The error in the product is much larger in absolute value than the original error.

> *Example 78:* Define A = 3, B = 3.00 000 003, C = B – A, and D = (B – A) * (1E +10). The displayed values for A and B are the same as the entered expressions. From these, the expected values are C = 3E – 8 and D = 300. But the displayed values are C = 2.79396772E – 08 and D = 279.396772. The error in C is due to loss of significance. The error in D is the result of magnification of the error in C by the large factor 1E +10.

The next example shows that the form of the algebraic expression used to define a function can affect the error in calculated function values.

> *Example 79:* For this example, press ₵-F to change to Parametric format and ₵-M to change to the Array display. Then, define
>
> X(T) = T * (1 – T/(T + 1))
>
> and
>
> Y(T) = T/(T + 1).
>
> Notice that the expression for X(T) simplifies to the expression for Y(T). These two functions should have the same values for all T. Also, as T gets very large, T/(T + 1) tends to 1; so the values of both X(T) and Y(T) should tend to 1.
>
> To test these conjectures, press
>
> ₵-T and enter 10 for Delta-T,
>
> ₵-A to generate an Array, and
>
> → to increase the value of T.
>
> For relatively small values of T (between 100 and 1000), the values of X(T) and Y(T) agree in the first 5 or 6 digits and appear to be tending to 1.
>
> Now press T, and enter T = 1E + 8. Notice that Y(T) = .999999999, which is very close to 1. But the values for X(T) are now greater than 1 and accurate in only the first 3 digits. For example, X = 1.00117177.
>
> Press T again, and enter T = 1E + 9. The values for X(T) are getting worse as T is increased. If you try T = 5E + 9, X(T) is greater than 2.3, and Y(T) is rounded to 1. Notice that the actual value of T/(T + 1) is never 1.
>
> Finally, try T = 5.72662305E + 9. The values of X(T) suddenly change from 2.66666666 to 0, and Y(T) = 1.

The values of X(T) greater than 2 are caused by loss of significance in the subtraction 1 – T/(T + 1) as T/(T + 1) gets very close to 1. This error is then magnified by multiplying by large T in T * (1 – T/(T + 1)) and producing X(T) = 2.66666666. Eventually, however, the value of T/(T + 1) is rounded to exactly 1, so that 1 – T/(T + 1) has value 0. Thus X(T) = 0.

The following example shows that it is often preferable to use T * T rather than T^2 in algebraic expressions.

Example 78: A comparison of X(T) = T * T and Y(T) = T^2 shows that T * T is often calculated more accurately than T^2. To see this, define X(T) and Y(T) as indicated and press

T to assign T = 0,
⌘-T to assign Delta-T = 1,
⌘-A to generate a table of values, and
→ to increase the value of T.

As T increases, the value of T^2 becomes inaccurate.

Negligible Addition

If a small number and a relatively large number are added, the effect of the small number on the sum might be negligible. This means that the effect is the same as that of adding zero to the larger number.

Example 81: In this example, we add the small values of 1/(2^X) to 1 and to 1000 to observe negligible addition. Press

⌘-F to restart in Function format,
⌘-M to change to the Array display,
F and define F(X) = 1 + 1/(2^X),
X and define X = 0,
⌘-X and define Delta-X = 1,
⌘-A and select F to produce an array, and
→ to scroll the table and observe the values X and F(X).

As X increases, 1/(2^X) gets very small, so the sum 1 + 1/(2^X) approaches 1. Scroll X up to X = 32. Notice that after X = 27, the function values are all 1. This is the result of negligible addition, not underflow. When X > 27, 1/(2^X) is so small that it has no effect when added to 1.

Now, press

Esc to return to the ? prompt,
F and define F(X) = 1000 + 1/(2^X),
X and define X = 0, and
⌘-A and select F to generate an array.

Again, scroll the table using → and observe the values of X and F(X). This time, the addition becomes negligible when X = 17. After X = 17, the function values are all 1000. The value of 1/(2^X) is too small, relative to 1000, to have an effect on the sum.

It is important to notice that the relative size of terms determines whether or not negligible addition occurs. In our example, for 17 < X < 27, 1/(2^X) has an effect when added to 1, but not when added to 1000.

Press Esc to return to the **?** prompt on the Array display.

Error Propagation

If computations are carried out with values that already are in error, the results are affected by these original errors as well as by errors introduced by the current computation. The effects of the original errors are called *propagated errors.* They would occur even if the current computation procedure itself produced no errors.

Error propagation can be a serious problem during the performance of a long sequence of computations. Even though each individual computation introduces only a relatively small error, the propagated errors from a large number of computations can have serious effects on the final results.

Example 82: A simple example of error propagation occurs on the Graphics display. As values are projected from a graph, the increment, Delta-X, is added over and over to the previous value of X. If Delta-X contains rounding errors, the error accumulates as the number of repetitions is increased. Press

- ⌘-R to restart,
- F and define F(X) = X,
- ⌘-X and enter .1 for Delta-X,
- ⌘-P to begin plotting the graph of F, and
- → to project larger values of X.

Starting at X = 0, the successive values of X are .1, .2, .3, . . . This is expected with Delta-X = .1. But soon (near 3.6), the X-values displayed are not exactly correct. For example, 3.9999999 appears where you expect 4. If you press ← to move X back to 0, you may find that 0 is not displayed. Rather, a very small positive value appears.

These errors occur because the increment, .1, cannot be stored exactly in your computer. (Remember that numbers are displayed in decimal form but stored in binary form.)

To remove the accumulated error from the current value of X, simply press X and change the current value. For example, change X from 3.99999999 to 4. When you press Return, the updated value of X is projected. Try it.

To avoid the propagation of errors in X, use a value for Delta-X that is stored exactly. For values of Delta-X less than 1, use 1/2, 1/4, 1/8, 1/16, ... Powers of 1/2 are binary numbers and are stored exactly. Since there is no error in the value of Delta-X used, there is no error propagation from this source. For example, press X and enter X = 0. Then press ⌘-X and enter 1/16. Now, when you scroll the projections, the displayed values of X are exact.

Press Esc to exit the Project/Plot option and end this example.

Avoiding Error Propagation

The difficulties with Delta-X encountered in the last example can affect options other than Ó-P, Project/Plot. For example, when Ó-G, GRAPH Function is used to graph a function, F(X), the first function value, is calculated at the left endpoint of the current interval. Then, each time a new F(X) value is calculated, X is incremented by the amount L/N, where L is the length of the current interval and N is one less than the current number of points being plotted. It is often a good idea to have L/N = 1/2, 1/4, 1/16, ...

On the default coordinate system, the interval, [-5,5], has length 10, so the numbers of points 21, 41, 81, 161, 321, . . . work well. The corresponding ratios, L/N, are 1/2, 1/4, 1/8, 1/16, 1/32, ... These binary numbers for the increment, L/N, avoid accumulated errors in X. This can be useful when you want the plotting option to use exact values.

For example, if you are using Ó-G, Graph Function to plot the graph of F(X) = X/(X * X – 4), the option should try to reject the exact values 2 and -2 at which the asymptotes occur. You should use 161 points. (See Example 30.) If you use Ó-P, Project/Plot to plot the graph, change Delta-X to 1/16, and start with X = 0.

To avoid error propagation, you should try not only to use accurate values to begin with, but also to keep the number of computations small (if possible).

For more techniques to minimize errors due to fixed precision arithmetic, consult your favorite numerical analysis textbook.

Appendix 11: Installing Your Own Help Screens and Text Displays

The Auxiliary Disk that you received with your copy of *Graphics Calculator* has only one Help screen on it. Using choice 4—Help Screen Editor—on the initial menu, you can build and install as many additional Help screens as you want on any copy of this Auxiliary Disk. The only limit is disk storage space. This means you can customize Help screens for your own purposes. You can also transfer Help screens from one disk to another. For example, you can transfer the program-disk Help screens to an Auxiliary Disk.

You can use Help screens as full-screen text displays in presentations and demonstrations that you conduct interactively or with O-Sequences. The demonstrations on the Demonstration Disk you received with the *Graphics Calculator* package use this technique extensively. Since each O-Sequence can have its own associated Help screens, you can use a number of different sets of Help screens on the same disk. For example, you can use O-Sequences, specific setups, and full-screen text displays to design tutorial modules that begin with automatic presentations of examples and techniques. When the O-Sequences end, the viewer can continue interactively with your set of Help screens, perhaps designed especially for the topic at hand. When *Graphics Calculator* presents your Help screen, the ? prompt input window is always positioned at the bottom of the screen. The viewer can make entries as directed by the Help screen or can make any other valid entry. *Graphics Calculator* will continue to access your set of Help screens until another O-Sequence is run, or until another ⌘-R, Restart is used.

Help Screen Filename Conventions

When you press ⌘-H or Esc, *Graphics Calculator* searches for a Help screen on the disk in the current disk drive, according to the following conventions:

1. If you have not run an O-Sequence since booting or restarting, the program looks for a file with the name Help.*n*, where *n* is a positive integer. For example, if you press ⌘-H and enter 3, the program tries to load Help.3. If it cannot find Help.3, it loads Help.1. After loading Help.3, if you press →, the program looks for Help.4. If you press ←, it looks for Help.2. Whenever a file is not found, Help.1 is loaded. To avoid error messages, every disk must have a Help.1 file. Although not required, it is a good idea to name Help screens in sequential order—Help.1, Help.2, Help.3, . . .
2. If you are running, or have run, an O-Sequence since booting or restarting *Graphics Calculator*, the program first looks for files with the same name as the O-Sequence, but with .*n* appended, where *n* is a positive integer. For example, if you are running, or have run, an O-Sequence with the name TEST, *Graphics Calculator* will look for TEST.3 when you or the O-Sequence request Help.3. If TEST.3 is not found, the program looks for Help.3, and eventually for Help.1.

If you run several O-Sequences consecutively (perhaps by chaining), the O-Sequence used last determines the filenames of the Help files that are loaded.

Making and Editing Help Screens

To build and save your Help screens on an Auxiliary Disk, use choice 4, Help Screen Editor, on the initial *Graphics Calculator* menu. Create your screens by typing them in, and save them with a name of your choosing. The Help Screen Editor has facilities for editing previously saved Help screens, checking disk catalogs, and using inverse or flashing video.

After you start the Help Screen Editor, insert into the disk drives the Auxiliary Disk(s) whose Help screens you wish to copy or modify. Notice the list of options on the bottom two lines of the screen. The character A represents the Ӧ key. For example, **[A]-Q(uit)** means that, to quit, you should press Ӧ and Q. Here is a summary of these and other keys and their purposes:

Key	Purpose
←, →, ↑, ↓	position the cursor at any location on the top 22 lines of the screen
Esc	end current entry before completion, or exit Filer
[A]-F(iler)	load, save, or delete a Help screen, or view a disk catalog
[A]-Q(uit)	exit to the initial *Graphics Calculator* menu
[A]-L(ine truncate)	erase from cursor to end of line
[A]-S(creen truncate)	erase from cursor to end of screen
[A]-V(ideo)	toggle video from Normal to Inverse to Flash
Return	terminate a line, or complete a filename entry
Space Bar	leave a space or erase a character

To type or edit text on the top 22 lines of the screen, use the four cursor keys to move the cursor around the screen. To load a previously saved Help screen for editing, press Ӧ-F, select LOAD, and enter its filename. When you press Return, the screen is loaded, and

you can modify it as you wish. If you try to load a file that is not a previously saved Help screen, the results are unpredictable.

To look at the Auxiliary Disk catalog before loading, saving, or deleting a file, press Ó-F and choose "CATALOG."

To save a screen, press Ó-F, select "SAVE," and enter the filename. You can use any temporary filename you want, but remember that, to be accessed by *Graphics Calculator,* filenames must follow the conventions explained above.

The current Help screen is not erased until you load another one, so you can continue to edit it after you save one version. Just press Esc to resume editing. To start a new screen, erase the entire current one. To do this, move the cursor to the upper left-hand corner of the screen and press Ó-S.

You can use normal, inverse, or flashing characters. The current choice is determined by the word "V(ideo)" at the lower left of the screen. If the characters are inverse or flashing, that is the current setting. Inverse and flashing characters are automatically converted to upper-case.